ADAM ŻUCHOWSKI

Szczecin University of Technology
Institute of Control Engineering
70-126 Szczecin, Poland
e-mail: chlod@we.ps.pl

DYNAMIC MEASUREMENT OF THE CURVE $y(x)$ DEFINED BY PARAMETRIC RELATIONS $x(t)$, $y(t)$ UNDER RANDOM DISTURBANCES

Analog *XY*-recorders constructed as servomechanisms with sliding motion have been practically out of use for some time. Drawing the curve $y(x)$ defined by parametric relations $xz(t)$, $yz(t)$ under additive, random disturbances is done by using measuring sensors with negligible dynamic properties, A/D converter card, computer and a printer, with a different time scale and with elimination of the influence of the dynamics of these devices . The influence of disturbances can be significantly lowered either by using special filters before storing the data in computer memory or by suitable processing of the data already stored in the computer memory. In the first case we have to choose an appropriate filter type and its time scale factor T, and in the second case - an appropriate processing mode in order to get the resulting curve $Y(X)$ as close as possible to the theoretical curve $y(x)$. Both techniques provide different possibilities and create different problems which require separate considerations.

Key words: measurement of the curve $y(x)$, disturbance filtration, signal processing

1. INTRODUCTION

The problem of dynamic measurement of the curve $y(x)$ defined by parametric relations $x(t)$, $y(t)$ was discussed in [1]. Since then the measurement technique has changed considerably which fully justifies reconsideration of the problem The measurement track of each component quantity $x(t)$, $y(t)$ consists of a sensor with negligible dynamics, possibly, a filter attenuating disturbances, since at the sensor outputs we get signals $x_z(t) = x(t) + z_x(t)$, $y_z(t) = y(t) + z_y(t)$, an A/D converter card (usually, the same one for both channels) and computer memory. The use of a printer with a suitable software allows to obtain the curve $Y(X)$ in time different from the real one but with elimination of the dynamics of the recording itself. There are two possibilities: the first one which relies on filters lowering the disturbance influence but at the same time introduces some deformations of $x(t)$ and $y(t)$, and the second one which does not use filters but replaces them by suitable processing of the disturbed signals $x_z(t)$ and $y_z(t)$stored in computer memory and in which disturbance attenuation only slightly deforms the curve $Y(X)$ with respect to the ideal curve (measurand) $y(x)$. Since processing is done by the “batch” method in time different than the actual one there is a possibility of using specific methods which are more complex and in some circumstances more effective in comparison with the use of filters. We will discuss both above methods.

2. USING FILTERS WORKING IN REAL TIME

Let us assume that signals $x_z(t)$ and $y_z(t)$ provided by the measuring sensors and bearing disturbances, are first filtered and then stored in computer memory. Since selection of optimal filter transmittances requires not easily accessible information and complicated calculations then it seems to be justified to consider the possibility of preliminary selection of a filter type, close to optimal, followed by an adjustment of the filter time scale factor T according to current conditions. The problem stated in this way can be solved using simplified dynamics models based on the Taylor series expansion of the convolution integrand function [2].

2.1. Simplified models of dynamics

The convolution of the filter input signal $x(t)$ (or $y(t)$) and the output signal $X(t)$ (or $Y(t)$) can be expressed in the form:

$$X(t) = \int_0^t k(v)x(t-v)dv = \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} m_i(t)x^{(i)}(t), \tag{1}$$

if we expand term $x(t-v)$ in the Taylor series in the neighborhood of t and introduce the notion of the i-th moment of the filter impulse response $k(t)$ as:

$$m_i(t) = \int_0^t v^i k(v)dv . \tag{2}$$

For all signals $x(t)$, continuous for $t > 0$, the series (1) is convergent and for "smooth" signals, with decreasing share of higher order derivatives $x^{(i)}(t)$, it can be replaced by the sum of a few initial terms. Setting:

$$m_0(t) = h(t), \quad x(t-t_0) = \sum_{i=o}^{\infty} \frac{(-1)^i}{i!} t_0^i x^i(t), \tag{3}$$

we obtain models of the form:

$$X(t) = h(t)x[t-t_0(t)] + \sum_{i=2}^{\infty} R_i(t)x^{(i)}(t) + \int_0^t k(t-v)z_x(v)dv,$$
$$Y(t) = h(t)y[t-t_0(t)] + \sum_{i=2}^{\infty} R_i(t)y^{(i)}(t) + \int_0^t k(t-v)z_y(v)dv, \tag{4}$$

where $h(t)$ is the step response of the filter,

$$t_0(t) = m_1(t)/m_0(t), \qquad R_i(t) = \frac{1}{i!}[m_i(t) - m_0(t)t_0^i(t)], \tag{5}$$

$z_x(t), z_y(t)$ are disturbances occurring in tracks x and y, where we also assumed that both filters are identical.

It is worth to point out the following facts:

If in expressions (4) we omit terms with higher order derivatives $x^{(i)}(t), y^{(i)}(t)$ for $i \geq 2$, and disturbances, then the delay $t_0(t)$ does not deform the relation $Y(X)$ in comparison to $y(x)$ provided $h(t) \cong 1$ with sufficient accuracy, i.e., after the settling time t_u of the filter. Since changes of $t_0(t)$ last much longer than changes of $h(t)$ it is recommended that $t_0(t)$ in both measuring tracks is the same and this justifies the use of two identical filters. Thus the conditions of appropriate filtration have the form:

$$h(t) \cong 1, \quad R_i(t) \cong 0, \quad |\int_0^t k(t-v)z(v)dv| \leq e, \tag{6}$$

where e is a sufficiently small number, and can be used to select a filter and its time scale factor T.

2.2. Filter selection criteria

Let us initially assume that filter is a linear element with the transmittance:

$$K_f(sT) = \frac{1 + b_1(sT) + b_2(sT)^2 + ... + b_m(sT)^m}{1 + a_1(sT) + a_2(sT)^2 + ... + a_n(sT)^n}, \tag{7}$$

where T is the time scale factor and $m < n$.

The first condition of appropriate filtration $h(t) \cong 1$ is satisfied after the settling time t_u defined by the inequality:

$$|h(t) - 1|_{t \geq t_{u\delta}} \leq \delta, \tag{8}$$

where the quantity δ is chosen arbitrarily. In our consideration we assume $\delta = 0{,}05$ and the settling time $t_{u\delta}$ is found by means of simulations. Since it is proportional to the factor T we can assume $T = 1$ and calculate the ratio $t_{u\delta}/T$. This time should be as small as possible since for $t < t_{u\delta}$ the curve $Y(X)$ is drawn in a different time scale in comparison to the curve $y(x)$.

The second condition of appropriate filtration is to keep a required level of disturbances. Exact computations require the knowledge of the disturbances power spectral density. In the simplest and at the same time bad conditions we can assume that disturbances are characterized by a minimal frequency Ω_2 and overall gain Z. The filter should attenuate the gain to the admissible level equal to eT. This means that we have to satisfy the condition:

$$|K_f(j\Omega_2 T)| \leq e, \tag{9}$$

which allows to find the required minimal value $\Omega_2 T$ for arbitrary assumed value of e (in our considerations we assume $e = 0.1$). The filter also deforms useful signals $x(t)$ and $y(t)$. If the frequency band of these signals is $(0, \Omega_1)$, then we may require to fulfill the condition:

$$|K_f(j\omega_1 T) - 1|_{\omega \leq \Omega_1} \leq \delta, \tag{10}$$

which allows to find maximal value of the product $\Omega_1 T$.

The smaller is the ratio Ω_2 / Ω_1 the narrower is the frequency band where filter damping is too low for disturbances and too high for the useful signal. Thus the small value of the ratio Ω_2 / Ω_1 can be regarded as a possible criterion of the filter selection. Short settling time $t_{u\delta}$

can be another criterion. Since it depends on the time scale *T* and the filter has to attenuate disturbances, then it is necessary to satisfy the condition (9) and consequently to use the index $(t_{u\delta}/T)(\Omega_2 T)$ as a quantity independent of the time scale *T*.

The deformation of the useful signal can be assessed in another simple way. Since the output signal of the measuring track is approximately [3] equal to the average value of the output signal over the time interval $(-T_u, 0)$, where $T_u \cong 2T(a_1 - b_1)$, then we can minimize deformation by minimizing the product $\frac{1}{2}T_u\Omega_2$ as independent of the time scale *T*. This slightly different criterion than the mentioned product $t_{u\delta}\Omega_2$. We can also minimize the value $R_i(t)$, e.g., for $i = 2, 3, 4$, assuming that the recorded signals *x*(*t*) and *y*(*t*) are smooth. Calculation of $R_i(t)$ is difficult and leads to very complicated formulas so in practice we have to restrict ourselves to the analysis of the value $R_i(\infty)$. The following formulas hold:

$$R_i(\infty)/T^i = R_i/T^i = \frac{1}{i!}(m_i - m_1^i)$$

and:

$$\begin{aligned}
&m_1 = a_1 - b_1,\\
&m_2 = 2![a_1(a_1 - b_1) - (a_2 - b_2)],\\
&m_3 = 3![(a_3 - b_3) - a_2(a_1 - b_1) - a_1(a_2 - b_2) + a_1^2(a_1 - b_1)],\\
&m_4 = 4![-(a_4 - b_4) + a_1(a_3 - b_3) + a_2(a_2 - b_2) + a_3(a_1 - b_1) -\\
&- 2a_1a_2(a_1 - b_1) - a_1^2(a_2 - b_2) + a_1^3(a_1 - b_1)].
\end{aligned} \tag{11}$$

In order to make the value R_i/T^i independent of the time scale we have to use the product $R_i\Omega_2^i$ and remember that $R_i(t)\Omega_2^i$ achieves its steady-state value after a long time of the order $(2...3)\,t_{u\delta}$, and the maximum of $R_i(t)$ may be larger than $R_i(\infty)$. For this reason criteria of the form $R_i\Omega_2^i = minimum$ are less useful, however the smaller the values of these indices the better.

2.3. Survey of properties of some typical filters

From the point of view of the mentioned criteria some typical filters (Butterworth, Tchebyshev, Legendre) are analyzed together with some filters in the form of time lag systems of *n*-th order and "Pade filters" of order *m*, *n* built as Pade approximations of the non-deforming transmittance $\exp(-sT_0)$, i.e., approximations by rational functions with *m*-th order nominators and *n*-th order denominators [4]. In each case the filter transmittance is transformed to the overall form (7) with $a_n = 1$ and with preservation of the condition (10), essential for Tchebyshev and Legendre filters. Some Pade filters do not satisfy this condition and have been rejected. The corresponding collection of performance indices obtained during simulations is presented in Table 1.

Table 1. Collection of performance indices for filters discussed in the paper.

Filter type	$\Omega_1 T$	$\Omega_2 T$	Ω_2/Ω_1	$\Omega_2 t_u$	$\Omega_2 T_u$	$R_2\Omega_2^2$	$R_3\Omega_2^3$	$R_4\Omega_2^4$	$\Omega_2 R_4/R_3$
I - 1	0.33	9.95	30.2	29.8	9.95	49.5	821	9390	11.4
I - 2	0.23	3.00	13.1	14.2	6.00	9.00	72.0	351	4.88

I - 3	0.19	1.91	10.2	12.0	5.72	5.46	38.2	154	4.03
I - 4	0.16	1.47	9.13	11.4	5.88	4.32	29.7	114	3.94
I - 5	0.14	1.23	8.54	11.3	6.15	3.78	26.4	101	3.83
B - 2	0.57	3.15	5.50	9.22	4.46	0.00	14.8	115	7.77
B - 3	0.69	2.15	3.11	12.8	4.30	0.00	3.31	14.3	4.32
B - 4	0.76	1.78	2.35	12.2	4.64	0.00	2.02	9.39	4.65
B - 5	0.80	1.58	1.98	12.1	5.12	0.00	1.63	8.38	5.14
B - 6	0.83	1.47	1.76	15.8	5.67	0.00	1.49	8.39	5.63
P - 1.2	0.61	8.21	13.5	33.2	20.1	0.00	0.00	2.27	
P - 0.3	0.67	2.16	3.24	12.1	3.93	0.00	0.00	11.9	
P - 1.3	0.72	2.70	3.76	11.6	7.80	0.00	0.00	31.8	
P - 2.3	0.75	7.71	10.2	71.7	27.6	0.00	0.00	0.00	
P - 1.4	0.96	2.08	2.16	14.1	6.87	0.00	0.00	0.00	
P - 2.4	0.81	2.70	3.33	19.6	11.8	0.00	0.00	0.00	
C - 2	0.89	3.21	3.59	16.9	3.71	3.41	33.8	138	4.08
C - 3	1.11	2.29	2.06	16.6	4.39	2.31	18.1	91.2	5.04
C - 4	1.30	2.01	1.54	18.9	4.26	2.16	18.9	81.1	4.29
C - 5	1.38	1.90	1.38	15.8	5.68	2.83	28.2	168	5.96
L - 3	0.58	2.26	3.90	11.2	4.43	1.78	12.8	62.9	4.91
L - 4	0.71	1.92	2.72	11.2	4.67	0.00	2.48	23.0	9.27
L - 5	0.90	1.81	2.02	15.2	5.39	0.90	7.42	3.18	0.43

I - k: time lag filters of order k, B - k: Butterworth filters of order k, P - i, j: Pade filters, C - k: Tchebyshev filters of order k, L - k: Legendre filters of order k.

It is difficult to determine a priori which of these indices are more important than the others. However, it seems that we should reject filters for which values of $R_4\Omega_2^4$ are large, e.g. larger than 100, and also filters for which index $\Omega_2 t_{u\delta}$ is large, e.g. we have rejected cases where its value exceeded 15. We will analyze now filters of the type B-3, B-4, B-5, P-0.3; P-1.3; P-1.4; L-3 and L-4.

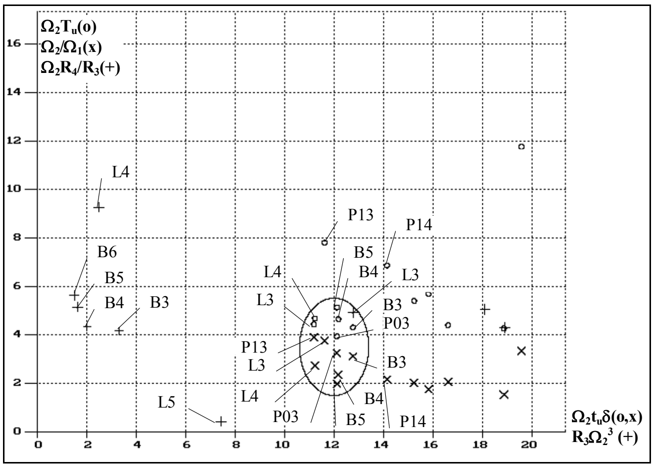

Fig. 1. Coordinates of points determined by filter performance indices.

In Fig. 1 we mark points corresponding to various coordinates as follows: $(\Omega_2 t_{u\delta}, \Omega_2 T_u)$ - circles, $(\Omega_2 t_{u\delta}, \Omega_2 / \Omega_1)$ - rotated crosses, and $(R_3\Omega_2^3, \Omega_2 R_4 / R_3)$ - crosses, for selected filters and as an illustration for a few rejected filters. Good accumulation is shown by points of filters B-3, B-4, B-5, L-3, L-4, P-0.3; and P-1.3 – their range is bounded by an ellipsoid and it seems that these types of filters should be taken into consideration, including the simplest filters. A collection of parameter values of such filters is presented in Table 2.

Table 2. Collection of parameter values for selected filters.

Filter type	a_1	a_2	a_3	a_4	b_1
B - 3	2.000	2.000	1.000	0.000	0.000
B - 4	2.613	3.414	2.613	1.000	0.000
P - 0,3	1.817	1.651	1.000	0.000	0.000
P - 1,3	2.163	2.079	1.000	0.000	-0.721
L - 3	1.961	1.575	1.000	0.000	0.000
L - 4	2.431	2.955	1.955	1.000	0.000

2.4. Simulations

The useful signal has been assumed in the form $x(t) = -\exp(-0.1t)\cos 0.5t$, $y(t) = \exp(-0.1t)\sin 0.5t$ with pseudorandom disturbances and the minimal frequency $\Omega_2 = 5.1\,\mathrm{s}^{-1}$. The ideal curve $y(x)$ and the curve with disturbances are shown in Fig. 2a. After applying a filter of the type L-4 with the time scale T chosen according to the proposed principles we obtained the following graphs shown in Fig. 2b: ideal curve $y(x)$, curve $Y(X)$without disturbances with the use of filtered denoted as $y(x) + L4$, and the graph $Y(X)$ with disturbances with the use of filter denoted as $y_z(x_z) + L4$. The graphs $y(x) + L4$ and $y_z(x_z) + L4$ differ from each other only slightly which proves a high efficiency of the disturbance elimination by filters. Comparison of $y(x)$ and $y(x) + L4$ shows the influence of time scale change at the initial stage, when $t < t_u$. Graphs in Fig. 2c show $y(x)$ without disturbances, and $y_z(x_z)$ with the use of filterers L-4, P-1.3 and also the worse filters: B-2, I-1, I-2 and I-4, with parameter T chosen according to the assumed principles. These graphs deflect much more from the ideal graph (especially, in the case of time lag filters I-1, …, I-4) which fully justifies the our choice.

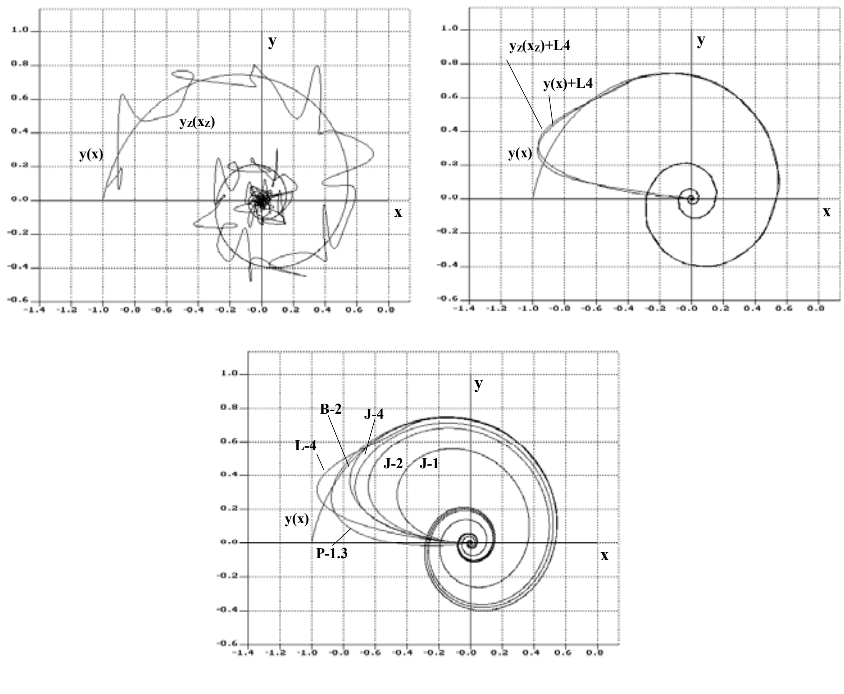

Fig. 2. Ideal curve $y(x)$ and simulation results for $Y(X)$obtained from disturbed measurements by means of various types of filters.

3. “BATCH” PROCESSING OF DISTURBED SIGNALS $x_z(t)$ AND $y_z(t)$

Recording of signals $x_z(t)$ and $y_z(t)$ in the computer memory allows for their “batch” processing, without restrictions imposed by filters working in real time, i.e. according to the causality principle. As a fundamental operation which attenuates disturbances we can use averaging of signals [5] with the even weight function $g(t)$ satisfying the condition:

$$g(t) = g(-t) \tag{12}$$

and the normalizing condition:

$$\int_{-d}^{d} g(t)dt = 1, \tag{13}$$

with:

$$x_z(t_0)_g = \int_{-d}^{d} x_z(t_0+v)g(v)dv, \quad y_z(t_0)_g = \int_{-d}^{d} y_z(t_0+v)g(v)dv. \tag{14}$$

Taking into account that $x_z(t) = x(t) + z(t)$, assuming continuity of the function $x(t)$ and expanding $x(t_0+v)$ into a Taylor series we obtain:

$$x_z(t_0)_g = x(t) + \sum_{i=1}^{\infty} \frac{1}{(2i)!} m_{g2i} x^{(2i)}(t) + \int_{-d}^{d} z(t_0+v)g(v)dv, \tag{15}$$

where:

$$m_{g2i} = \int_{-d}^{d} v^{2i} g(v)dv, \tag{16}$$

because the function $g(t)$ is even (see (12)). This type of averaging does not introduce delays and in the case of different signals $x_z(t)$, $y_z(t)$ we can use different weight functions $g_x(t)$, $g_y(t)$ which are also called measurement windows. The norming condition (13) implies that the weight functions have to depend on the width of the averaging interval $(-d, d)$ and hence also their moments m_{g2i} depend on d. For a given type of the weight function we have to choose parameter d in such a way that, e.g. the averaged value of the disturbance component with minimal frequency Ω_2 is sufficiently small (condition (3)). For this purpose it convenient to use logarithmic plots of the spectrum of windows $G(j\omega)$, for which we define the so-called bandwidth and the roll-off rate. As a selection criterion for the type of window we can assume a minimal value of the product $d\Omega_2$ together with minimal values of moments m_{g2i} and the condition on appropriate filtration of disturbances, the quotient of the bandwidth Ω_1 and the band Ω_2, defined as for usual filters, etc. The measurement windows have rich literature [6], [7] and their discussion does not seem necessary within this context. However, it is worth to point out the possibility of applying another method of processing signals $x_z(t)$ and $y_z(t)$, leading to graph $Y(X)$ close to the ideal $y(x)$.

3.1. Looking for $x(t)$ and $y(t)$ as solutions of linear differential equations

Let us assume that functions $x(t)$ and $y(t)$ are solutions of linear differential equations:

$$\sum_{i=0}^{p} a_i x^{(i)}(t) = A, \qquad \sum_{i=0}^{q} b_i y^{(i)}(t) = B, \tag{17}$$

with initial conditions $x^{(i)}(0),\ y^{(i)}(0)$. For different types of $x_z(t)$ and $y_z(t)$ we can assume the constants A and B either equal to zero or ± 1, as needed.

If the used weight function satisfies the additional condition:

$$g^{(i)}(-d) = g^{(i)}(d) = 0 \text{ for } i = 0,1,...,n-1,$$

where $n-1 \ge p$, $n-1 \ge q$, then due to the relations:

$$x^{(i)}(t_0)_g = (-1)^i \int_{-d}^{d} x(t_0+v) g^{(i)}(v) dv ,(18),$$

which follows from:

$$x^{(i)}(t_0)_g = \int_{-d}^{d} x^{(i)}(t_0+v) g(v) dv ,$$

by integrating by parts and the formula (18), we can find derivatives of disturbed signals $x_z(t)$ and $y_z(t)$ without differentiation of disturbances (and the signals themselves) [5]. Hence in Eq. (17), after averaging, all terms except the coefficients a_i and b_i will be known. These coefficients may be found by minimizing the quadratic performance indices [8]:

$$J_x = \int_{d}^{t_m-d} [\sum_{i=0}^{p} a_i x_z^{(i)}(t_0)_g - A_g]^2 dt_0 ,$$

$$J_x = \int_{d}^{t_m-d} [\sum_{i=0}^{q} b_i y_z^{(i)}(t_0)_g - B_g]^2 dt_0 , \tag{19}$$

which leads to simple, linear systems of algebraic equations for a_i and b_i. Once they are computed, for arbitrarily assumed p, q and d, it is possible to find $J_{x_{\min}}(p,d), J_{y_{\min}}(q,d)$ and select such p, q, d, which give the best results. In this way we can find the forms of differential Eq. (17). If the models (17) are good (small values of J_x and J_y), then to obtain right functions $x(t)$ and $y(t)$ without disturbances it suffices to calculate initial conditions of the Eq. (17) and solve them by any simulation method. In this way we avoid deformation of $Y(X)$ in the initial stage for small t and gain the possibility of prediction for large times not covered by measurements.

3.2. Calculation of the initial conditions

Neither signals $x_z(t)$ and $y_z(t)$ nor the averaged signals $x_z(t_0)_g$ and $y_z(t_0)_g$ are useful for calculations of required initial conditions $x^{(i)}(0),\ y^{(i)}(0)$ and the reason is that they are either disturbed or deformed by the operation of averaging. Nevertheless, these conditions can be found, with sufficient accuracy, by means of the following method:

The solution of the linear differential Eq. (17) has the form:

$$x(t) = x(0)f_0(t) + x^{(1)}(0)f_1(t) + ... + x^{(p-1)}(0)f_{p-1}(t) + Af_a(t) \tag{20}$$

and similarly for the function $y(t)$. For assumed a priori initial conditions $x^{(i)}(0)$ and the value of A the equation can be solved by means of any simulation method and this leads to the relation:

$$x(t_0)_g = x(0)f_0(t_0)_g + x^{(1)}(0)f_1(t_0)_g + ... + Af_a(t_0)_g . \tag{21}$$

Repeating the same operation for p+1 different combinations of initial conditions $x^{(i)}(0)$ and A for the same t_0 we obtain a system of linear algebraic equations which can be solved for $f_0(t_0)_g$, $f_1(t_0)_g$,..., $f_{n-1}(t_0)_g$, $f_a(t_0)_g$. These computations should be carried out for different values of t_0 from the interval $(d, t_m - d)$, where t_m is the observation time of $x_z(t)$.

In turn, for a computable value of $x_z(t_0)_g$ we have the equation:

$$x_z(t_0)_g = x(0)f_0(t_0)_g + x^{(1)}(0)f_1(t_0)_g + ... + x^{(p-1)}(0)f_{p-1}(t_0)_g + Af_a(t_0)_g , \tag{22}$$

in which the initial conditions are unknown. Minimizing the performance index:

$$J_{xw} = \int_d^{t_m-d} [x_z(t_0)_g - x(0)f_0(t_0)_g - x^{(1)}(0)f_1(t_0)_g - ... - x^{(p-1)}(0)f_{p-1}(t_0)_g - Af_a(t_0)_g]^2 dt_0 , \tag{23}$$

for previously calculated functions $f_0(t_0)_g$, …, $f_a(t_0)_g$ we obtain a system of linear algebraic equations for unknown initial conditions, with an additional decrease in the influence of disturbances. These calculations have to be carried out separately for each function $x_z(t)$ and $y_z(t)$, preferably with the use of suitable auxiliary programs. After the initial conditions $x^{(i)}(0)$ and $y^{(i)}(0)$ are computed, we obtain from equations (17) the required functions $x(t)$ and $y(t)$ on arbitrary time interval starting from $t = 0$ and suitable curves $Y(X)$ close to the curve $y(x)$. Minimal values of indices J_{xw} and J_{yw} allow to assess the accuracy of these calculations.

3.3. Simulation of experiment

The functions $x(t)$ and $y(t)$ considered in Section 2 correspond to solutions of the differential equation:

$$x^{(2)}(t) + 0.2x^{(1)}(t) + 0.26x = 0 ,$$

with the initial conditions $x(0) = -1$, $x^{(1)}(0) = -0.1$ and:

$$y^{(2)}(t) + 0.2y^{(1)}(t) + 0.26y = 0 ,$$

with the initial conditions $y(0) = 0,\ y^{(1)}(0) = 0.5$. If we now consider disturbances as in the previous example in Section 2 and use the described identification procedures, then we obtain equations:

$$x^{(2)}(t) + 0.2004x^{(1)}(t) + 0.2600x = 0,$$

$$y^{(2)}(t) + 0.193y^{(1)}(t) + 0.256y = 0,$$

corresponding to minimal values of the performance indices $J_{x\min}$ and $J_{y\min}$ for $d = 2$s. The initial conditions for these equations, found by means of the described method, practically coincided with the actual conditions with the error less than 1%. The obtained curves $Y(X)$ together with $y(x)$ and $y_z(x_z)$ are shown in Fig. 3. They are much more accurate than those obtained by means of good filters.

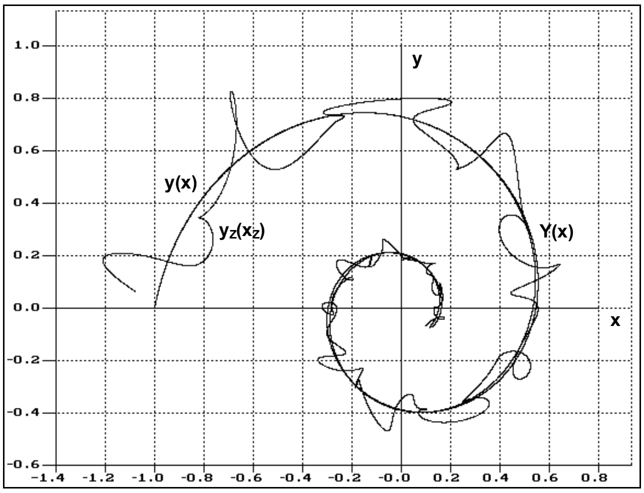

Fig. 3. Graphs of $y(x)$, $y_z(x_z)$ and $Y(X)$ obtained by the method of processing signals $x_z(t)$ and $y_z(t)$ described in Section 3.1.

4. SUMMARY

The simulation results confirm the practical effectiveness of both methods. In the case of the first method there is a possibility of finding even better filters, in particular for modified values of the parameters δ and e (assumed arbitrarily) and admissible levels of the used performance indices. By changing these factors we can influence deformations of the signals $x(t)$ and $y(t)$ The situation may be also improved by using filters with time-varying parameters [9], but it should be emphasized that these filters are more complicated.

The second method and in particular search for linear differential equations describing signals $x(t)$ and $y(t)$ requires to use specialized computer software [10, 11], however it seems that this pays off. Obviously, not all functions $x(t)$ and $y(t)$ correspond to solutions of linear differential equations but definitely this is the case for smooth functions described by polynomials in t with a finite number of terms.

NOTATION

$a_{i,} b_i$ - parameters of transmittances or differential equations,

A, B	- constant functions equal to ± 1 or 0,
$2d$	- interval of averaging,
e	- tolerance band,
$f(t)$	- function of time,
$g(t)$	- weight function,
$h(t)$	- filter step response,
J_x, J_y	- quadratic performance indices,
$K_f(s)$	- filter transmittance,
$k(t)$	- filter impulse response,
$m_i(t)$	- i-th order moment of the impulse response,
m_{g2i}	- $2i$-order moment of the weight function,
$R_i(t)$	- filter sensitivity function,
s	- operator,
t, v	- time,
T	- filter time scale factor,
$t_0(t)$	- delay time of filter dynamics model,
$t_{u\delta}$	- time of the transient process,
T_u	- time of averaging,
ω, Ω	- frequencies,
$x(t), y(t)$	- useful signals,
$X(t), Y(t)$	- recorded signals,
$x^{(i)}(t), y^{(i)}(t)$	- time derivatives of $x(t)$ and $y(t)$,
$z(t)$	- disturbances.

REFERENCES

1. Żuchowski A.: *On measurements of curves y(x) defined by parametric relations x(t) and y(t) with disturbances*. PAK 1/1988 (in Polish).
2. Żuchowski A.: *On passing of slowly varying signals through linear systems*. PAK 2/1982 (in Polish).
3. Żuchowski A.: *Dynamic measurements*. Textbook, Szczecin University of Technology, Szczecin 1984 (in Polish).
4. Halawa J.: *Methods of calculating simplified transmittances and their applications in automatic control and power engineering*. Politechnika Wrocławska, Instytut Cybernetyki Technicznej, Prace Naukowe 1991, Seria Monografie Nr 21 (in Polish).
5. Kordylewski W., Weck J.: *Averaged differentiation of disturbed measured signals*. PAK 6/1988 (in Polish).
6. Nuttal A.H.: *Some windows with very good sidelobe behaviour*. Speech and Signal Processing 29 (1981), No. 1, p. 84-91.
7. Piotrowski J.: *Measurement applications of signal analysis*. PWN, Warsaw 1991 (in Polish).
8. Boćkowska M., Żuchowski A.: *On a certain method of finding parameters for simplified dynamics models*. PAK 12/1994 (in Polish).
9. Kaszyński R.: *Low-pass filters and constant component filters with varying parameters*. Prace Naukowe Politechniki Szczecińskiej Nr 569, Instytut Automatyki Przemysłowej Nr 19, Szczecin, 2001 (in Polish).
10. Boćkowska M., Żuchowski A.: *Method and computer program IDO for identification of parameters of simplified, linear dynamics models and its measurements application*. Materiały IV Sympozjum "Modelowanie i Symulacja Systemów Pomiarowych", Krynica 1994, Wyd. Zakładu Metrologii AGH, Kraków 1994 (in Polish).

DYNAMICZNY POMIAR CHARAKTERYSTYKI XYZ OKREŚLONEJ PRZEZ PARAMETRYCZNE ZALEŻNOŚCI XXX, ZZZ PRZY ZAKŁÓCENIACH PRZYPADKOWYCH

Streszczenie

Analogowe rejestratory *XY* budowane jako serwomechanizmy z ruchem ślizgowym praktycznie wyszły z użycia. Wykreślanie charakterystyki $y(x)$ określonych zależnościami parametrycznymi $x_z(t)$, $y_z(t)$ przy obecności addytywnych, losowych zakłóceń odbywa się przy wykorzystaniu czujników pomiarowych o pomijalnych własnościach dynamicznych, karty przetwornika A/C, komputera oraz drukarki, w innej skali czasu z eliminacją wpływu dynamiki tych urządzeń. Wpływ zakłóceń można wydatnie zmniejszyć bądź stosując specjalne filtry jeszcze przed zapisem przebiegów w pamięci komputera, bądź dokonując odpowiedniego przetwarzania sygnałów zakłóconych zapisanych już w pamięci komputera. W pierwszym przypadku należy dobrać odpowiedni typ filtru i jego współczynnik skali czasu T, a w drugim - odpowiedni typ przetwornika tak, by uzyskany przebieg $Y(X)$ był możliwie bliski przebiegowi teoretycznemu $y(x)$. Obie techniki oferują odmienne możliwości i stwarzają problemy wymagające oddzielnego omówienia.