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OSCILLATORY MECHANICAL FLOWMETER - THEORETICAL ANALYSIS OF OPERATION

The paper proposes a method consisting in theoretical description of the main forces and related moments acting on a flowmeter oscillator. The mathematical analysis carried out has led to an equation representing the oscillator movement wherein phenomena occurring when a viscid stream flows around a movable body are taken into consideration.

Key words: stream mass, oscillatory flowmeter, movement equation

1. INTRODUCTION

The aim of theoretical description of oscillatory mechanical flowmeter is to find an unique relationship between the flow stream and the vibration frequency of the oscillator. Such a relationship determines the flowmeter processing function, thus it describes its metrological properties. The issue is confused by complicated geometry of the flow space; the time-variable boundary conditions caused by oscillator movement with respect to the stream, axially symmetrical flow of the stream and three-dimensional flow around a body need to be considered.

The constancy of the *St* number (at about 0.18) is noticeable, for instance, in flow round patterns for axially symmetrical bodies starting from some value of the *Re* number (of the order of $2x10^4$) and this is put into use in vortex flowmeters (Fig. 1) [1], [2].

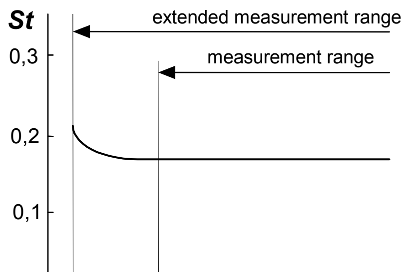

Fig. 1. Measuring range of vortex flowmeters.

As seen from a metrological point of view, if the *St* number is constant, the flow stream can be determined by measuring a frequency proportional to the average stream velocity being specific for a particular processing coefficient. In the case of vortex flowmeters this is the frequency of vortex generation whereas for oscillatory mechanical units - the frequency of oscillator vibrations. For the latter flowmeters, the linearity of relation between frequency and velocity, $f = f(w)$, is also decisive for their measuring range. When an appropriate correction is introduced at extremities (most often at the low speed end), the measuring range may be extended. In case a compressible stream measurement is considered, an additional correction for density variations needs to be introduced [3].

In case of oscillatory mechanical flowmeters, the limit of the measuring range is not determined by the phenomenon of a vortex path created downstream the generator (as is the case for vortex units), but first and foremost, it is dependent on the geometry and dynamic properties of the oscillator applied and the way it is fixed. This impedes the theoretical description of flowmeter operation, being however also advantageous as it allows the flowmeter measuring range to be extended at low extremity for *Re* around 2.500 by appropriate selection of constructional features.

2. MEASURING SYSTEM

In order to become acquainted with the construction and operating principle of the oscillatory mechanical flowmeter under consideration including a Hall-effect vibration frequency sensor, the unit is shown in Fig. 2, whereas Fig. 3 illustrates its main constructional components: a stream divider (3) and an oscillator (5) [4].

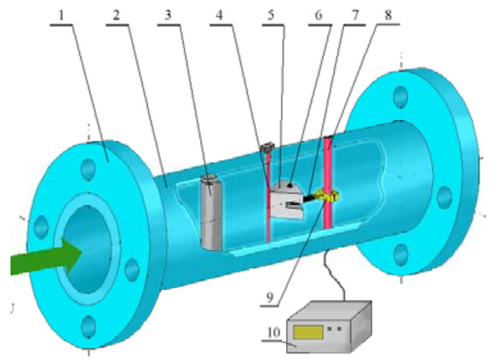

Fig. 2. Oscillatory flowmeter with Hall-effect frequency sensor: 1 - flange, 2 - housing, 3 - stream divider, 4 - oscillator axle, 5 - oscillator, 6 - magnet, 7 - Hall-effect plate, 8- support, 9 - adjusting pin, 10 - counting unit.

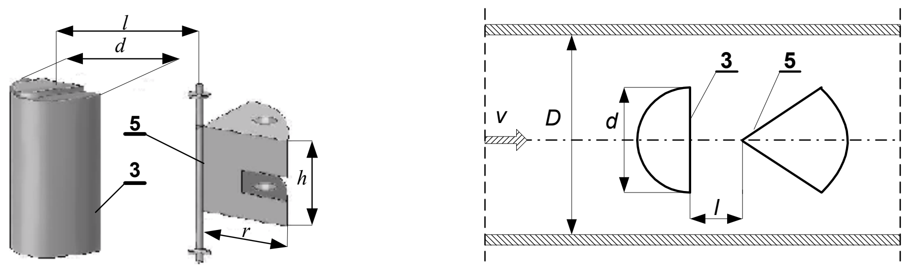

Fig.3. Constructional elements of the flowmeter: 3 - stream divider, 5 - oscillator, *l* - spacing between stream divider and oscillator, *d* - stream divider diameter, *D* - pipe internal diameter, *r* - oscillator radius, *h* - oscillator height.

Individual elements of the flowmeter under consideration have been installed in a prepared header which is inserted directly into the flow duct of air stream, the main components of which are (Fig. 3):

- a semi-cylindrical stream divider (3) mounted so that it is possible to move it along the duct axis, thus to vary the distance between the divider and the oscillator,
- an oscillator (5) fixed on its axis of rotation.

3. FORCES ACTING ON THE OSCILLATOR

An exact determination of active forces exerting pressure on the oscillator being flown araound by a viscid stream requires to resolve a set of movement equations and to define the time-variable distribution of pressures on the oscillator surface. These forces need to be known to derive the oscillator movement equation, and further on, basing on this equation - to find the flowmeter processing function [5]. Therefore, an attempt was made to consider determinable active forces and moments existing in the flow around the oscillator body. To this purpose, a model of flat flow round pattern for axially-symmetrical body (oscillator) restrained with incompressible fluid stream was used. Assumptions were made of rectangular velocity profile in the inflow stream and invariable cross-sectional area of the flow duct. An equilibrium of moments resulting from the forces acting on the oscillator was considered as it makes a partial turn around the central axle (point O in Fig. 4).

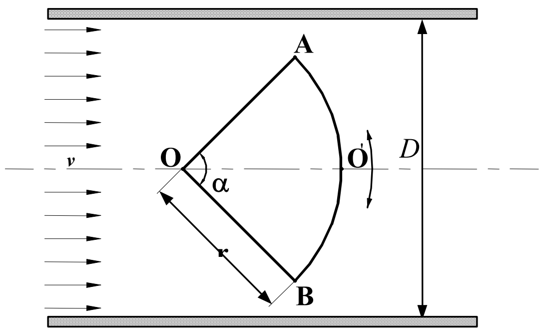

Fig. 4. Diagram of flow area.

The oscillator's thrust faces, OA and OB, are subject to dynamic pressure forces, the values of which are equal to each other in the initial position (shown in Fig. 4), i.e. when the axis OO' of the oscillator coincides with duct axis, and are given by:

$$N_d^{OA} = N_d^{OB} = \rho\, A\, v^2 \cos\left(\frac{\pi}{2} - \frac{\alpha}{2}\right) = \rho\, r\, h\, v^2 \sin\frac{\alpha}{2}, \qquad (1)$$

where: ρ - liquid density, A - thrust face surface area (side surface of oscillator), r - oscillator radius, h - oscillator height (in the plane normal to that shown in Fig. 3).

When the oscillator swings from its initial position (initial disturbance) by an angle φ (Fig. 5), it causes the dynamic force on one thrust face to increase and the respective force on the other face to decrease.

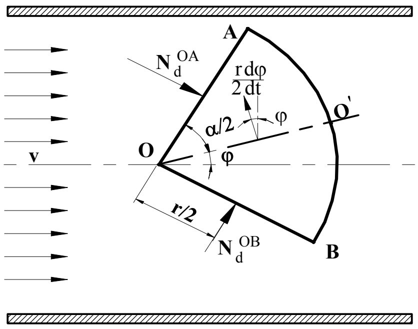

Fig. 5. Dynamic thrust forces in case of oscillator movement.

Thus, a resultant force and resultant moment will occur and it will strive to restore the initial position of the oscillator. One reason why such moment appears is the change in stream thrust angles to individual faces. However, if this reason alone is considered, this would lead to an equation of a conservative system wherein the oscillator generates non-decaying free oscillations the amplitude and phase of which are determined only by initial conditions having been set out. On the other hand, a change of the dynamic thrust on the side surface in transient flow around oscillator is related to the change in speed of the stream against this surface when we consider that, during movement, it is the sum of inflow velocity and linear speed of oscillator rotary movement. Thus, on the side surface of the oscillator, which moves at the moment in the direction opposite to the stream (the surface approaching the duct wall), the relative thrust velocity will be higher than that for a stationary oscillator, while on the surface which moves in conformity with the stream, the relative velocity will be accordingly lower.

Finally, upon allowing for the variability of the angle φ and the inflow velocity, the dynamic thrust forces for a specific swing of the oscillator (while assuming, as a first approximation, that the dynamic forces act along the arm of $r/2$) can be given by:

$$N_d^{OA} = \rho\, r\, h \left(v + \frac{r}{2}\frac{d\varphi}{dt} sin(\frac{\alpha}{2} + \varphi) \right)^2 sin(\frac{\alpha}{2} + \varphi)\,, \tag{2a}$$

$$N_d^{OB} = \rho\, r\, h \left(v - \frac{r}{2}\frac{d\varphi}{dt} sin(\frac{\alpha}{2} - \varphi) \right)^2 sin(\frac{\alpha}{2} - \varphi) \tag{2b}$$

and the resultant moment of these forces with respect to oscillator axis of rotation is:

$$M_d = \frac{\rho r^2 h}{2} \left(\left(v + \frac{r}{2}\frac{d\varphi}{dt} sin(\frac{\alpha}{2} + \varphi) \right)^2 sin(\frac{\alpha}{2} + \varphi) - \left(v - \frac{r}{2}\frac{d\varphi}{dt} sin(\frac{\alpha}{2} - \varphi) \right)^2 sin(\frac{\alpha}{2} - \varphi) \right). \tag{3}$$

The sign of the moment in (3) results from the direction of oscillator movement.

Consideration of a time-variable stream thrust velocity due to rotary movement of the oscillator represents a factor which causes suppression of the initial deflection. It appears then that the dynamic force in the deflection direction is growing faster (and respectively, the force in the opposite direction decreases slower) than in stationary flow round pattern. Although it is not a formal explanation of the existence of dissipation force causing the suppression, introducing the variable inflow velocity into the equation makes indirect allowance of variability of pressure resistance coefficient of oscillator body related to a change of its position with respect to the inflowing stream.

Another source of initial attenuation of oscillator initial deflection is the viscous friction resistance. More precisely, viscous friction forces act on each thrust face, OA and OB, and are also the result of the stream flowing around the oscillator. However, independently of the oscillator's position, these forces operate tangentially to its side surfaces and their operating line goes through its axis of rotation. Thus, despite the presence of a pair of friction forces in the flow round pattern, they cause no additional torque. On the other hand, we can consider the frictional resistance of the oscillator which causes a rotational movement in viscous liquid while treating the stream as brought momentarily to a standstill. This resistance will act on the oscillator according to its rotational direction and will depend on liquid density, on side surface area of the oscillator and on its linear velocity as in the case of turbulent flow around a body. When we assume that this resistance acts at the middle of the oscillator radius, the frictional resistance moment can be expressed as:

$$M_t = c_t \, sgn\left(\frac{d\varphi}{dt}\right)\frac{\rho\, r^2\, h}{2}\left(\frac{r}{2}\frac{d\varphi}{dt}\right)^2 , \qquad (4)$$

where: c_t - dimensionless coefficient of frictional resistance.

The sign function in this expression takes into account the direction of rotation and it was introduced because the remaining elements of Eq. (4) are always positive.

Additional attenuation of oscillator vibrations in the flowmeter causes mechanical friction determined by the method of fixing the oscillator in the duct. One can however expect that mechanical friction during oscillator movement affects its behaviour like viscous friction, so it makes no formal modification to the movement equation. Therefore, the model under description neglects (in the first approximation) the resistance in oscillator bearings. As it would appear, this factor may play a decisive part during oscillator start-up by determining the flowmeter starting threshold, hence the lower limit of its measuring range. Determination of how mechanical friction affects the oscillator movement is the task covered by a study of metrological properties of the flowmeter under consideration.

In the situation being analysed, the flow round pattern takes place in a space confined by the duct walls which causes border effects absent when a free stream is considered. When the oscillator approaches the duct wall, it causes a necking of the flow channel between the oscillator and the wall (Fig. 6), and as a consequence the pressure in this channel goes higher. A reverse situation exists on the opposite side of the oscillator, thus during oscillator swing the static pressure acting on its side surfaces changes periodically. The resultant force of these pressures supports the oscillator movement towards its initial position.

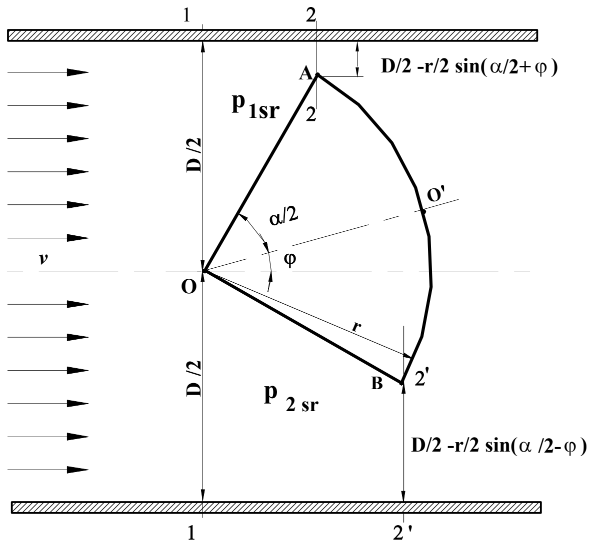

Fig. 6. Shape of the flow channel during oscillator deflection.

In the assumed two-dimensional flow around model, the pressure in the channel between the duct wall and the oscillator is in linear relationship with the channel width. If the pressure loss in this channel is neglected, the oscillator is subject to the difference of average dynamic pressures existing on its both sides. The average dynamic pressures between the 1-1 and 2-2 cross-sections of the duct are respectively (Fig. 6):

$$p_{1\,sr} = \frac{\rho\, v^2}{4}\left(\frac{D}{D + r\,sin\left(\frac{\alpha}{2}+\varphi\right)}+1\right), \tag{5a}$$

$$p_{2\,sr} = \frac{\rho\, v^2}{4}\left(\frac{D}{D - r\,sin\left(\frac{\alpha}{2}-\varphi\right)}+1\right). \tag{5b}$$

However, the moment of the pressure force acting on the oscillator is:

$$M_c = -\frac{\rho\, r^2\, h}{8}\, v^2\left(\frac{D}{D + r\,sin\left(\frac{\alpha}{2}+\varphi\right)} - \frac{D}{D - r\,sin\left(\frac{\alpha}{2}-\varphi\right)}\right). \tag{6}$$

The minus sign before this moment means that it is in opposite direction than the moments of dynamic thrust and viscous friction.

The forces specified above and acting on the oscillator during its movement represent a system of external forces. As the oscillator responds to applied forces, there appears the force of inertia which can be determined for specific dimensions and density of oscillator material. The moment of inertia acting on the oscillator during its rotary movement for the shape being considered (oscillator as L' of cylinder) may be given as follows:

$$M_b = I\frac{d^2\varphi}{dt^2} = m\frac{r^2}{8}\frac{d^2\varphi}{dt^2} = \rho_o V\,\frac{r^2}{8}\frac{d^2\varphi}{dt^2} = \frac{1}{32}\pi\,\rho_o\,r^4\,h\frac{d^2\varphi}{dt^2}\,, \tag{7}$$

where: I - oscillator solid moment of inertia, m - oscillator mass, V - oscillator volume, ρ_o - density of oscillator material.

The reasons causing self-excited vibrations are believed to be the phenomena which occur on the surface of the body being flown around and which change the pressure field around it. As a result, such a change in pressure distribution on the surface of the body being flown around shall lead to originating an additional force and its moment which acts toward oscillator movement and which compensates the attenuating forces.

The mechanism of energy transfer from the liquid stream to the body being flown around is related to flow separation from its surface [6]. For specific thrust velocity, it is the shape of the body being flown around which is decisive to causing the flow separation phenomenon and to the position of the flow separation point. A displacement of this point for a stationary body (being in static equilibrium) can be forced only by a change of the inflow stream velocity. In oscillatory flowmeters there is an additional movement of the flown around surfaces of the working element (oscillator) with respect to the liquid stream. Such a relative movement initiated by an incidental initial disturbance, in a certain range of flow streams, may cause the stream separation point to move from the position of the steady-state flow around, and consequently a change of pressure distribution on the surfaces being flown round and generation of an additional force, even if the stream velocity is constant. In case of the oscillator shape considered, where two flown round surfaces exist, OA and OB, the stream separation in the oscillator initial position, shown in Fig. 4, occurs at edges A and B . However during the movement, the flow around applies to the whole surface under movement with a horizontal component opposite to the stream velocity (OA surface in Fig. 6). On the other hand, the horizontal component of the velocity for OB surface will be, at the same time, directed consistently with the stream velocity. It seems probable that such consistency of directions combined with the change in position of the flown round surface with respect of the inflowing stream may be the reason of displacing the stream separation point from the B edge towards the oscillator axis of rotation, so consequently, only a part of OB surface will be flown around. The situation will reverse when the oscillator moves in the opposite direction. Partial flow around one of oscillator side surfaces will change the energy passed in the fluid stream - oscillator system as compared with the flow around without separation.

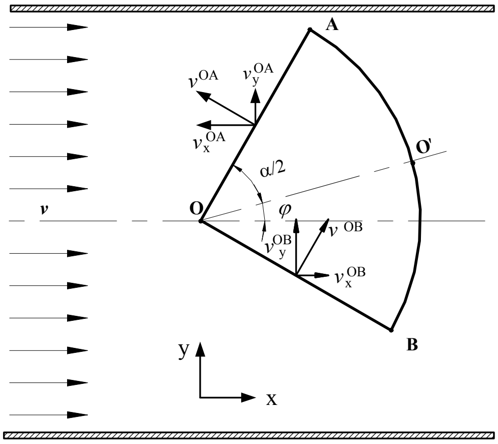

Fig.7. Components of linear velocity of the flown around surfaces.

Displacement of the separation point on one of the flown around surfaces, thus also the effect of such displacement, i.e. the appearance of an additional force, would be dependent on the angular speed of the particular surface. If we assume, simplifying the problem considerably, that the maximum of this additional force occurs at the maximum angular speed of the oscillator surface flown around, its effect on the oscillator will be delayed with respect to the dynamic thrust force because the largest dynamic thrust on the side surface exists at the maximum oscillator deflection while the largest angular speed is at zero deflection angle. Such delay prevents return of the oscillator to static equilibrium and maintains its periodic non-decaying vibrations.

Some analogy to oscillations of a body flown around by a viscid stream is represented by self-exciting vibrations caused by dry friction known for a long time.

If the additional force causing negative attenuation of the oscillator and being related to stream separation from its surface is made dependent on the square of inflowing stream velocity, on liquid density and area of the oscillator side surface (as these variables are decisive for other pressure forces acting on the oscillator), the moment of this force with respect the oscillator axis of rotation may be written as follows:

$$M_{dod} = -c_{dod}\left(\frac{d\varphi}{dt}\right)\frac{\rho\, r^2\, h}{2}\, v^2, \qquad (8)$$

where: c_{dod} - is the factor of proportionality dependent on the oscillator angular speed.

It is rather difficult to provide an accurate physical interpretation of the c_{dod} factor which, as pointed out in the formula (8), is the factor of proportionality dependent on the oscillator angular speed. Initially, using a great simplification, it can be treated as the ratio of the surface area being flown around by a stream to the total side surface of the oscillator. When assuming, as a first approximation, that the value of this factor rises according to a linear relation to the angular speed (i.e. to introduce linear relation between the position of stream separation point and the angular speed), then when we introduce the moment, M_{dod}, in the oscillator movement equation, we get the following movement equation:

$$\sum_i M_i = M_b + M_d + M_t + M_c + M_{dod} = 0,$$

$$\frac{1}{32}\pi\ \rho_o r^4 h \frac{d^2\varphi}{dt^2} + \frac{\rho\, r^2\, h}{2}\left(\left(v + \frac{r}{2}\frac{d\varphi}{dt}\sin(\frac{\alpha}{2}+\varphi)\right)^2 \sin(\frac{\alpha}{2}+\varphi) - \left(v - \frac{r}{2}\frac{d\varphi}{dt}\sin(\frac{\alpha}{2}-\varphi)\right)^2 \sin(\frac{\alpha}{2}-\varphi)\right) +$$
$$+ c_t\ \mathrm{sgn}\left(\frac{d\varphi}{dt}\right)\frac{\rho\, r^2\, h}{2}\left(\frac{r}{2}\frac{d\varphi}{dt}\right)^2 - \frac{\rho\ r^2\ h}{8} v^2 \left(\frac{D}{D + r\sin\left(\frac{\alpha}{2}+\varphi\right)} - \frac{D}{D - r\sin\left(\frac{\alpha}{2}-\varphi\right)}\right) - \qquad (9)$$
$$- c_{dod}\left(\frac{d\varphi}{dt}\right)\frac{\rho\, r^2\, h}{2} v^2 = 0.$$

As it results from Eq. (9), for specific range of the c_{dod} factor, the solution represents the function of oscillating motion with constant frequency independent of initial conditions. This is shown in Fig. 8 where exemplary solutions for an oscillator being flown around by an air stream are illustrated.

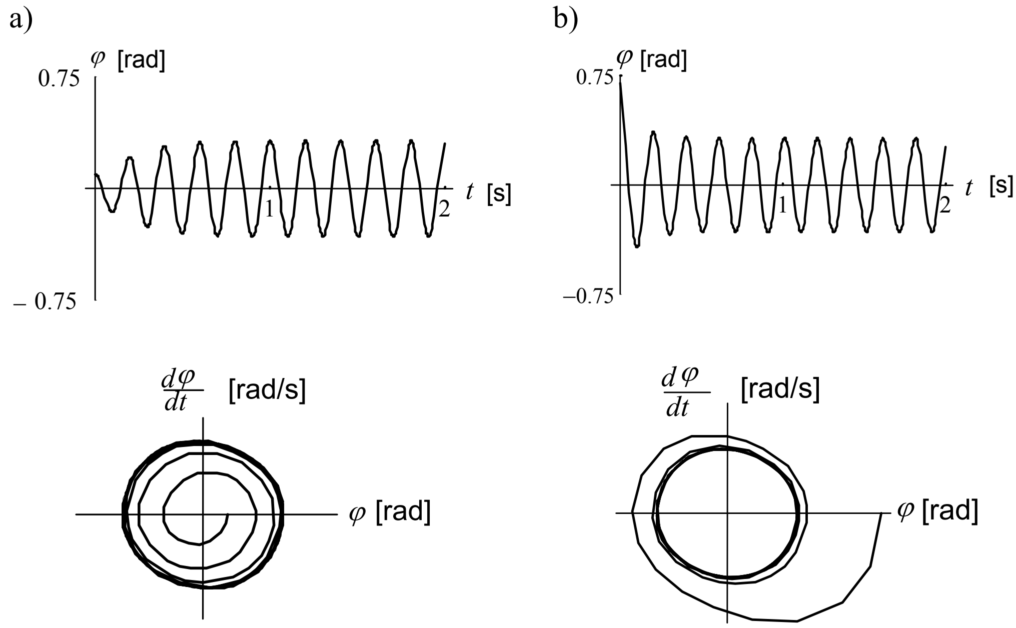

Fig. 8. Changes in oscillator deflection angle, in the time domain and in the phase plane:
a) small initial deflection; b) large initial deflection (higher than stable vibration amplitude).

In the example provided, a linear relation of $c_{dod} = f\left(\frac{d\varphi}{dt}\right)$ was assumed with factor of proportionality equal to 0.02 at changes of oscillator angular speed within (-10; 10) s^{-1}. Simulations refer to an air flow at the speed $v = 10$ m/s ($Re = 52000$). Stable oscillation vibrations for the additional moment were found for the factor c_{dod} within (0.005 ÷ 0.05). Smaller values of the factor cause attenuation of initial disturbance and return of the oscillator to static equilibrium, while larger values lead to an increase of vibration amplitude with time.

Simulations have been run for the model to determine the flowmeter characteristic in the form of a relation between the mass stream and oscillation frequency. The flow conditions (range of velocity and sensor parameters) and flowmeter geometry were assumed as the initial data. Thus, the characteristics have been found for air flow speeds up to 20 m/s in a duct with diameter D = 80 mm, wherein an oscillator with a radius of r = 30 mm was planned.

Figure 9 illustrates the comparison between the flowmeter characteristic for a processing constant found experimentally and its characteristic resulting from a numerical simulation based on the model.

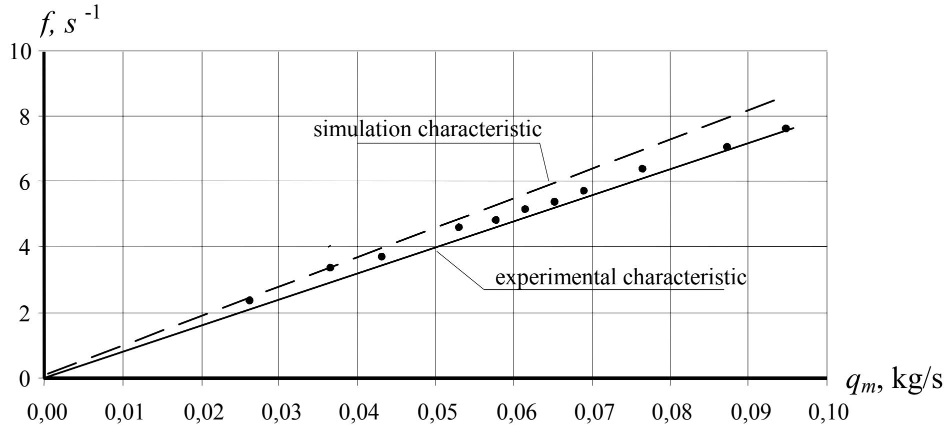

Fig. 9. Experimental and simulation characteristics of the flowmeter.

The consistency of characteristics shown in Fig. 9 seems to be promising due to further works on a mathematical model of the mechanical flowmeter. Non-decaying periodical vibrations of the oscillator appear in numerical solutions of the proposed movement equation for conditions encountered in technical applications and as seen, the characteristic of the oscillatory flowmeter determined from the model is of linear nature.

4. SUMMARY

An attempt to determine the movement equation and the results attained for model resolving are indicative of further research to find a physical verification and to obtain a more accurate form of the Eq. (9), both in qualitative aspects (e.g. by visualizing real flow around), and in the quantitative sense on the basis of measurement data. The main aim of the work was to develop a relatively simple mathematical model which, at the same time, would provide a sufficient description of the real properties of the oscillatory mechanical flowmeter. In practical terms, it would allow to estimate the effect of constructional features of the flowmeter on its operation without the need to resolve full movement equations for a viscous liquid. Introducing the Eq. (8) into Eq. (9) has caused that stable oscillations of the working element of the flowmeter appear in the mathematical model, which is in consistency with observations. The proposed mathematical model of the flowmeter needs however further development as concerns a more accurate physical interpretation and quantitative effects of the additional force introduced, and a more accurate description of how other external forces affect the oscillator. In these considerations, the effect of constructional dimensions on flowmeter operation shall be also included.

The results which have been attained confirmed the assumptions taken from theoretical considerations about the influence of constructional parameters on metrological properties of an oscillatory mechanical flowmeter, and it especially concerns the spacing between the oscillator and the stream divider as well as oscillator inertia.

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TEORETYCZNA ANALIZA PRACY OSCYLACYJNEGO PRZEPŁYWOMIERZA MECHANICZNEGO

Streszczenie

W artykule zaproponowano metodę, polegającą na opisie teoretycznym głównych sił i związanych w nimi momentów działających na oscylator przepływomierza. Przeprowadzona analiza matematyczna doprowadziła do wyznaczenia równania ruchu oscylatora, z uwzględnieniem zjawisk występujących podczas opływu strugą lepką ruchomego ciała.