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NUMERICAL ANALYSYS OF THE INFLUENCE OF ASYMMETRIC DEFORMATION OF THE VELOCITY FIELD ON METROLOGICAL PROPERTIES OF THE ORIFICE

A change of the stream direction during a flow through a single elbow or their system causes asymmetric deformation of the velocity field. If two elbows are located in various planes, also a stream swirl can occur. The influence of these two phenomena on the accuracy of measurement of the fluid stream with orifices has been so far determined experimentally. In this paper theoretical considerations have been applied. Fluid motion has been described by the Reynolds set of equations closed with the *k-ε* model of turbulence. The three-dimensional set of equations has been solved for a fluid flow through the straight pipeline in the flow system with one or two elbows located in various planes. The tests were performed in order to estimate the influence of the velocity field upstream the orifice on the pressure distribution on the wall and the discharge coefficient for different distances of local obstacles from the orifice. The calculation results have been compared with the available experimental results.

Keywords: flow measurements, pipe orifice, asymmetric flow, measurement error

1. INTRODUCTION

Correct measurements of the flow rate require a fully developed turbulent velocity profile in the section upstream the orifice [1]. The orifice location strongly influences the measuring accuracy. Local obstacles before the orifice cause disturbances of the stream velocity field which can cause a change of the discharge coefficient *C*, and - in consequence - serious measurement errors. The discharge coefficient *C* is defined by

$$C = \frac{q_m \sqrt{1-\beta^4}}{\frac{\pi}{4} d^2 \sqrt{2 \Delta p \rho}}, \tag{1}$$

where q_m is the mass flow rate, $\beta = d / D$ is the orifice diameter ratio, d is the orifice diameter, D is the pipe diameter, Δp is the pressure drop and ρ is the fluid density.

Many realized works concerned the determination of the pipe length necessary to obtain a fully developed turbulent flow after typical local obstacles and the influence of the stream disturbance on the flow ratio [2, 3, 4]. A wide review of investigations on the influence of local obstacles (also flow stabilizers) on measurements of the flow ratio with orifices is presented in [3]. In papers by Schröder [4], Alleon [5] and Nagel and Jaumotte [6], the authors analyzed the effect of distances between the orifices and typical local obstacles on the discharge coefficient *C* of the pipe orifice. From Alleon's tests [5] it results that it is difficult to eliminate asymmetric deformation of the stream and swirl caused by the presence of two elbows in various planes. Its influence on measuring accuracy is strong, even in the case of great distances from the orifice.

Irving [7] found a dependence of values and signs of ΔC on the orifice diameter ratio β. For an orifice with $\beta = 0.5$, changes of the discharge coefficient varied in the range from -3.5% to +0.75% for changes L in the range $2D \div 37D$. For orifices with $\beta = 0.8$ and the same changes of L, values of ΔC were included in the range from -3.25% to -0.75%. Similar results were obtained by Mattingly and Yeh [8]. Nagel and Jaumotte [6] obtained results quite different from those obtained by Irving [7], and the observed differences could result from another method of pressure difference measurement or different roughness of the applied pipes. Sattary [10] found that for $\beta = 0.57$ and $L = 23D$ there is no strong change of discharge coefficient C, and $\beta = 0.75$ at the distance $36D$ the change of the discharge coefficient is low and equal to $\Delta C = +0.25\%$.

The elbow 90° is a local obstacle causing asymmetric stream disturbances, which has been best known so far. From the test results it appears that such disturbance generates negative values of the discharge coefficient changes in a wide range of changes of the diameter ratios [9 - 13]. It has been found that the measuring error is directly proportional to the value of β and inversely proportional to the distance between the elbow and the orifice. Blake and al. [12] found that there was no relation between the measuring error and the Reynolds number.

A numerical analysis of the influence of the elbow and the system of two elbows on the change of the discharge coefficient C for an orifice with the diameter ratio $\beta = 0.8$ was presented in [17]. A negative change of the flow coefficient was observed for each case.

This paper contains theoretical considerations on the influence of asymmetric deformation of velocity fields and stream swirls caused by the presence of one elbow or a system of two elbows located in various planes on measurements of the mass stream with an orifice of $\beta = 0.5$.

2. MATHEMATICAL MODEL

Let us consider a flow of a viscous incompressible fluid through a flow system with an orifice. The flow systems are shown in Fig. 1. Fluid motion is described by a system of equations containing equations of motion

$$\rho \mathbf{U} \nabla \mathbf{U} = -\nabla p + \nabla \mu_{ef} \left(\nabla \mathbf{U} + \nabla \mathbf{U}^T \right) + \frac{2}{3} \rho \nabla k \tag{2}$$

and equation of continuity

$$\nabla \mathbf{U} = 0 , \tag{3}$$

where $\mathbf{U}$ is the mean velocity vector, ρ - fluid density, k - kinetic energy of turbulence, and $\mu_{ef} = \mu + \mu_t$ - effective viscosity, where μ - molecular viscosity.

The above system of equations has been closed with the semi-empirical model of turbulence based on the equations for kinetic energy of turbulence k and dissipation rate ε [15].

In the standard Launder-Spalding k-ε model, turbulent viscosity is calculated from

$$\mu_{ef} = \rho C_\mu \frac{k^2}{\varepsilon} . \tag{4}$$

Values of k and ε are calculated from

$$\rho \nabla k \mathbf{U} = \nabla \left(\mu + \frac{\mu_t}{\sigma_k} \right) \nabla k + G - \rho \varepsilon \,, \tag{5}$$

$$\rho \nabla \varepsilon \mathbf{U} = \nabla \left(\mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \nabla \varepsilon + \frac{\varepsilon}{k} \left(c_1 G - c_2 \rho \varepsilon \right). \tag{6}$$

In calculations, the following empirical constants are assumed

$$\sigma_k = 1.00 \;\; \sigma_\varepsilon = 1.30 \;\; c_1 = 1.44 \;\; c_2 = 1.92 \;\; c_\mu = 0.09$$

The system of Eq. (2) - (6) was solved by the finite element method [16]. In calculations, a straight interval of the pipeline with an orifice (Fig. 1a) and flow systems with one elbow 90° with radius of curvature 1.5*D* (Fig. 1b) and two elbows (Fig. 1c) were considered. The area was divided into three-dimensional isoparametric finite elements containing 8 nodes in the element nodes. For discretization, 50,000 elements were used for a straight section (Fig. 1a), from 80,000 to 400,000 elements were applied in the case of a single elbow (Fig. 1b), and in the case of two elbows (Fig. 1c) from 87,000 to 430,000 elements were used, depending onthe length of the straight interval of the pipeline upstream the orifice. The computer program FIDAP [18] was used in numerical computations.

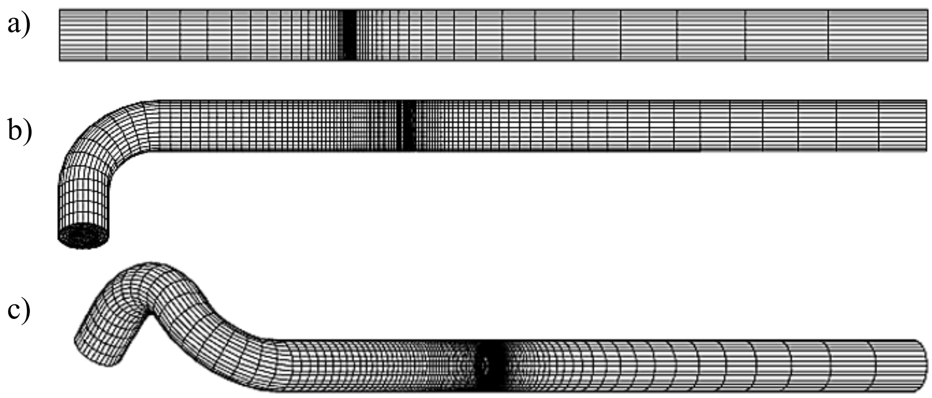

Fig. 1. Flow systems with orifices: a) without local obstacles, b) with one elbow, c) with two elbows in various planes.

In the inlet sections of the flow systems, the applied velocity distribution was described by the Prandtl equation

$$u(r) = u_{\max} \left(1 - \frac{r}{R} \right)^{1/7} , \tag{7}$$

where R is the pipe radius.
The kinetic energy of turbulence was dependent on the mean velocity

$$k = \frac{3}{2} I^2 u_{śr}^2 , \tag{8}$$

where I is the turbulence intensity.
Dissipation rate of kinetic energy of turbulence in the inlet section was calculated from the following equation:

$$\varepsilon = \frac{k^{3/2} C_\mu^{3/4}}{0.07R} . \tag{9}$$

When simulating turbulent flows using turbulence models, it is particularly challenging to model the viscosity affected near-wall regions. Wall functions are a collection of semi-empirical formulas and functions that in effect “link” the solution variables at the near-wall cells and the corresponding quantities on the wall.
The standard wall functions are based on the proposal of Launder and Spalding [15]. The law-of-the-wall for mean velocity yields

$$U^* = \frac{1}{\kappa} \ln\left(E y^*\right), \tag{10}$$

where

$$U^* \equiv \frac{U_P C_\mu^{1/4} k_P^{1/2}}{\tau_\omega / \rho} , \tag{11}$$

$$y^* \equiv \frac{\rho C_\mu^{1/4} k_P^{1/2} y_P}{\mu} \tag{12}$$

and $\kappa = 0.42$ is the von Karman constant, E = 9.793 is an empirical constant, U_P is the mean velocity of the fluid at point P, k_P is the turbulence kinetic energy at point P near the wall, y_P is the distance from point P to the wall.
The logarithmic law for mean velocity is valid for $y^* > 11.225$. When the mesh is such that $y^* < 11.225$ at the wall-adjacent cells, the laminar stress-strain relationship is $U^* = y^*$.
In the k-ε models, the k equation is solved in the whole domain including the wall-adjacent cells. The boundary condition for k imposed at the wall is

$$\frac{\partial k}{\partial n} = 0 , \tag{13}$$

where n is the local coordinate normal to the wall.
The production of k at the wall-adjacent point is computed from

$$G \approx \tau_\omega \frac{\partial U}{\partial y} = \tau_\omega \frac{\tau_\omega}{\kappa \rho C_\mu^{1/4} k_P^{1/2} y_P} \tag{14}$$

and ε is computed from

$$\varepsilon = \frac{C_\mu^{3/4} k_P^{3/2}}{\kappa y_P}. \tag{15}$$

3. RESULTS OF NUMERICAL CALCULATIONS

A series of numerical calculations was performed for a pipe of $D = 0.1$m in diameter with the orifice diameter ratio $\beta = 0.5$. The length of the straight pipe downstream the orifice was reduced to 10D. Calculations were performed in order to analyze the influence of velocity field deformation connected with the fluid flow through the elbow or a system of two elbows on the flow structure in the considered system.

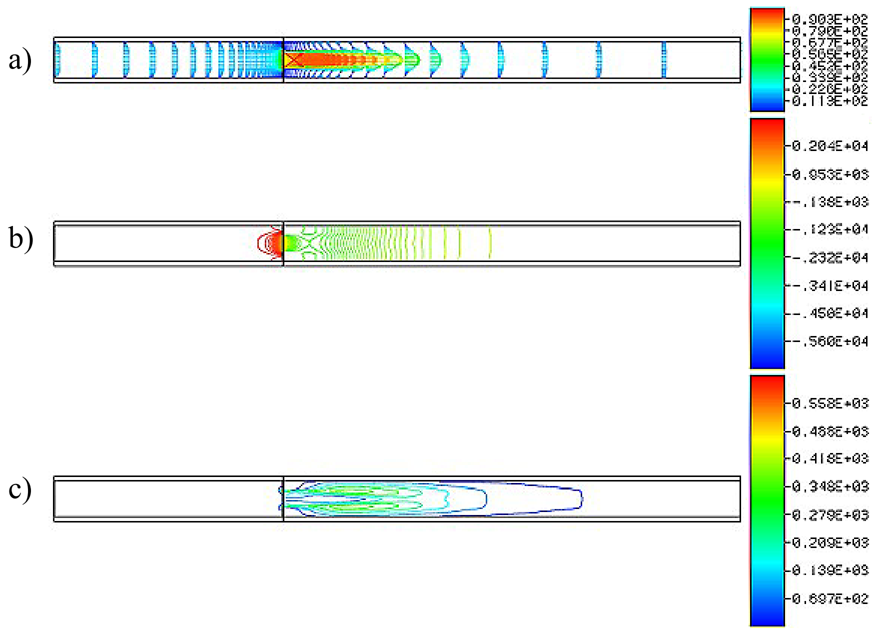

Fig. 2. Results of numerical calculations for a flow system without an elbow: a) velocity vectors, b) isobars, c) kinetic energy of turbulence.

The presented results correspond to the Reynolds number $Re = 90000$ based on the pipe diameter.

Fig. 2 shows the calculation results for the case when a fully developed turbulent flow takes place in the pipe cross-section upstream the orifice using a mesh shown in Fig. 1a.

Fig. 3 shows the results of calculations for a system with a single elbow with radius of curvature $1.5D$ occurring at the distance $5D$ upstream the orifice using a mesh shown in Fig. 1b.

The calculation results for a flow system with two elbows located in various planes are shown in Fig. 4 using a mesh shown in Fig. 1c. In both considered cases it was assumed that a fully developed turbulent flow occurs in the inlet section.

Velocity field deformation upstream the orifice influences the measuring results of the fluid stream. The change of the discharge coefficient ΔC was defined as

$$\Delta C = \frac{C_d - C_0}{C_0} 100\% , \tag{16}$$

where C_d is the discharge coefficient of the orifice under deformed velocity field and C_0 is the value for a fully developed flow.

During calculations, the authors considered variable lengths of the straight pipe interval upstream the orifice equal to 5D, 10D, 15D, 20D and 30D for an assessment of the influence of the distance of the obstacle on changes of the discharge coefficients.

Figures 5 ÷ 8 present the numerical calculation results. The calculated values of ΔC were compared with available experimental data.

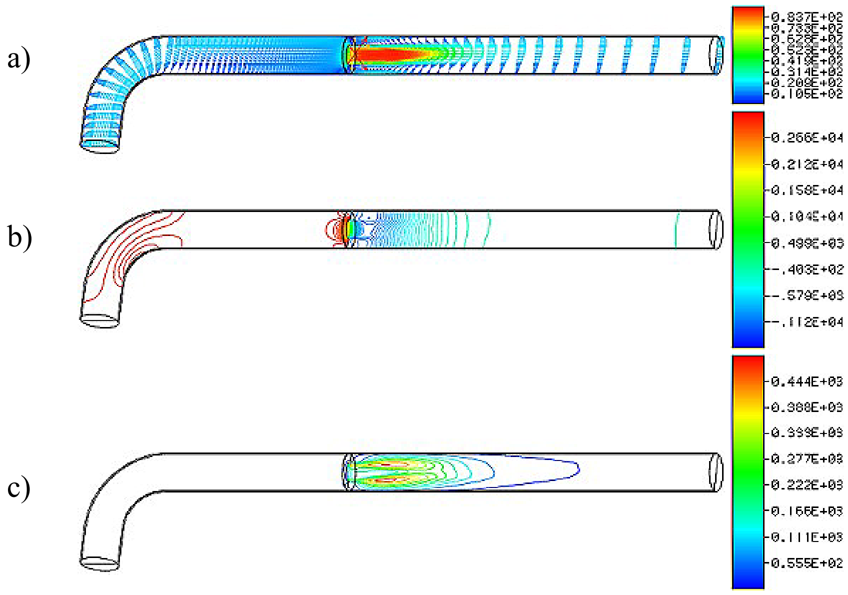

Fig. 3. Results of numerical calculations for a flow system with a single elbow: a) velocity vectors, b) isobars, c) kinetic energy of turbulence.

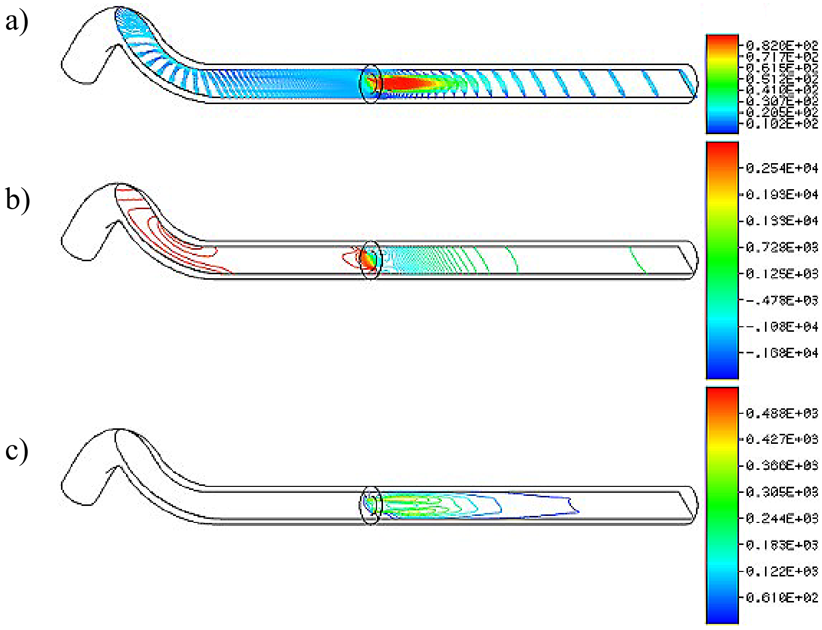

Fig. 4. Results of numerical calculations for a flow system with two elbows: a) velocity vectors, b) isobars, c) kinetic energy of turbulence.

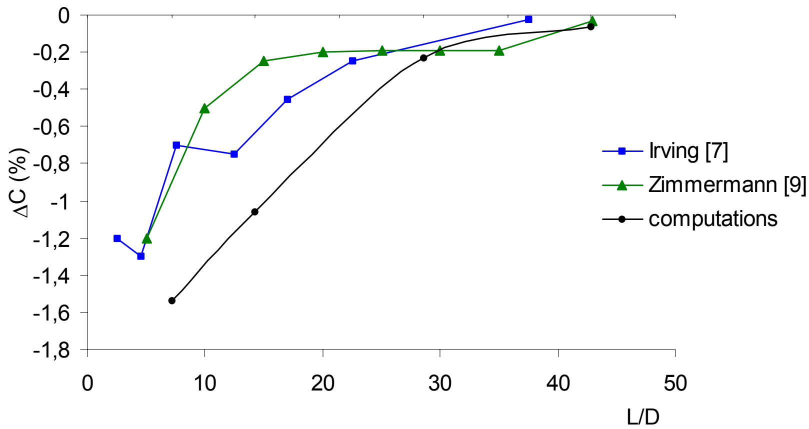

Fig. 5. Influence of the single elbow distance from the orifice on changes of the discharge coefficient for an orifice with corner taps.

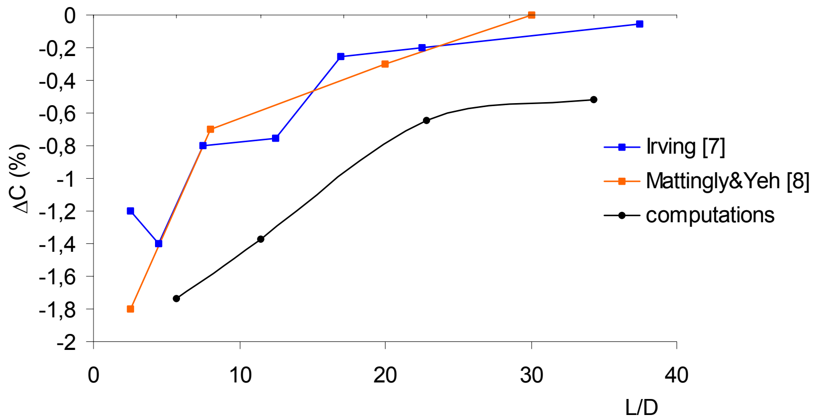

Fig. 6. Influence of the single elbow distance from the orifice on changes of the discharge coefficient for the orifice with the flange taps.

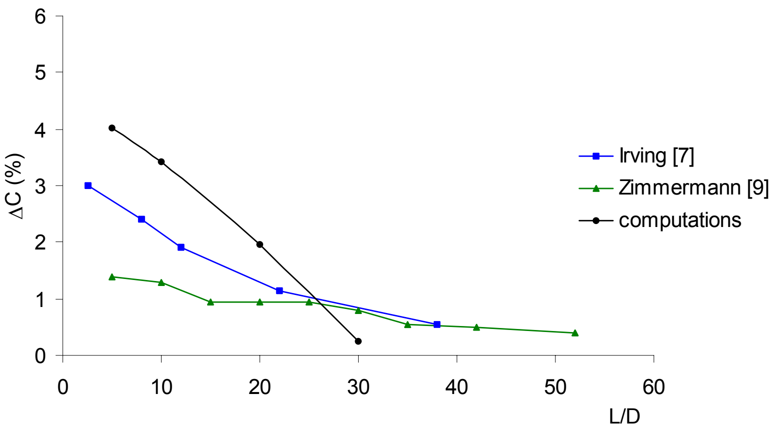

Fig. 7. Influence of two-elbow system distance from the orifice on changes of the discharge coefficient for an orifice with corner taps.

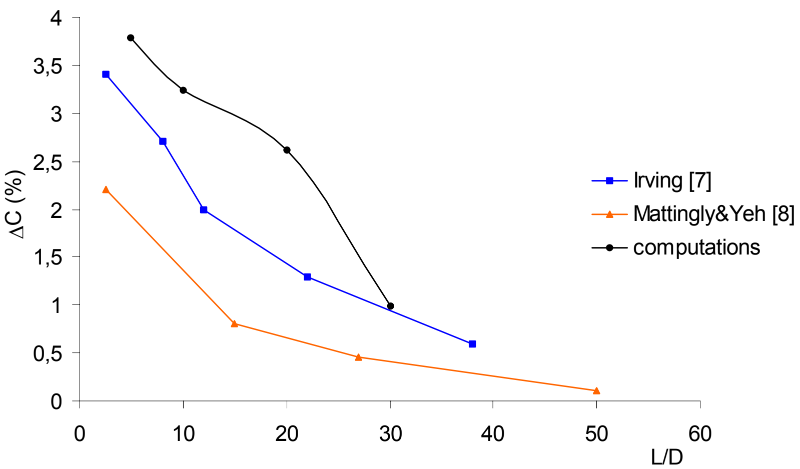

Fig. 8. Influence of the two-elbow distance from the orifice for changes of the discharge coefficient for an orifice with flange taps.

From the presented data it results that a single elbow located upstream the orifice causes a negative change of the discharge coefficient, and two elbows generate a positive change. In both cases an increase of the upstream straight length causes a drop of ΔC. Good qualitative conformity between calculations and experimental results can be observed. In Figures 9 the calculated velocity distributions along the pipe axis for the considered systems were compared. The observed small drop of velocity upstream the orifice for the system without elbows results from lack of coherence assumed in the inlet section of the boundary condition (Eqs (7) - (9)) with the velocity field generated by the equations of the mathematical model for a fully developed flow.

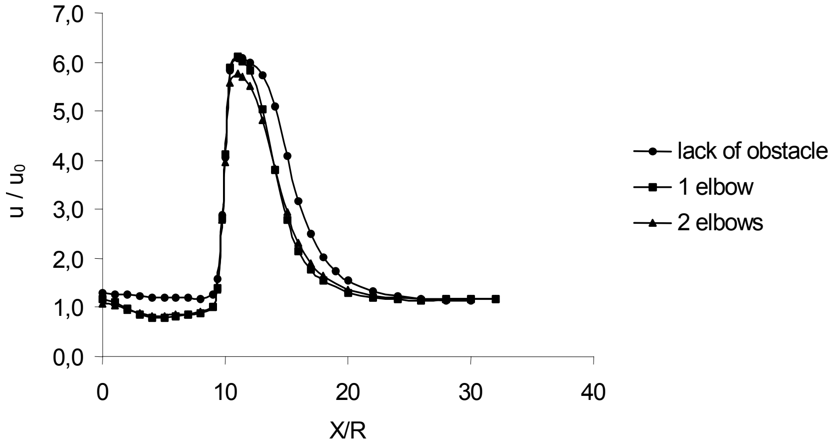

Fig. 9. Velocity distribution in the pipeline axis

The presence of an elbow or two elbows in the system causes a velocity decrease upstream the orifice and faster velocity decay after the orifice. Figures 10 and 11 shows pressure distributions on the axis and at the wall of the pipe.

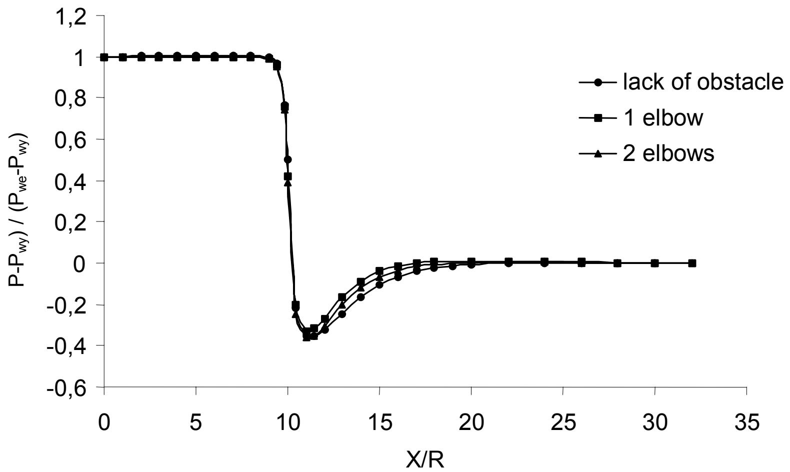

Fig. 10. Pressure distribution in the pipe axis.

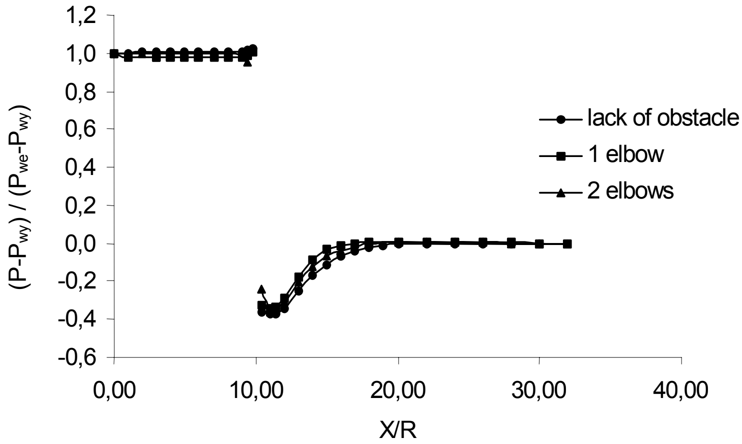

Fig. 11. Pressure distribution at the pipe wall.

Asymmetry of the velocity field influences the pressure distribution at the pipeline wall and causes changes of the discharge coefficient of the orifice.

4. CONCLUSIONS

A numerical analysis was performed for three complex cases of a three-dimensional flow in a pipe with the orifice. The calculations included asymmetric deformations of the velocity field connected with a fluid flow through the elbow or a system of two elbows in various planes. A system of two elbows generates not only asymmetric deformation of the velocity field but swirl of the stream as well. Both local obstacles cause a change of the discharge coefficient C which influences the accuracy of measurements. The system with one elbow generates a negative change of the discharge coefficient: $\Delta C < 0$, and two elbows - a positive change: $\Delta C > 0$. The results were compared with the available experimental data and their good qualitative conformity was found. The observed differences between numerical calculation results and experimental data can be connected with:

a) the number and arrangement of the numerical grid nodes near the orifice, which strongly the influence accuracy of calculations (see [14, 18]},
b) possible small accuracy of the applied turbulence model in case of three-dimensional swirled flows,
c) the assumed boundary conditions at the pipe wall for the turbulence model equations.

The main cause of observed differences should be explained in future investigations.

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ANALIZA NUMERYCZNA WPŁYWU ASYMETRYCZNEJ DEFORMACJI POLA PRĘDKOŚCI NA WŁASNOŚCI METROLOGICZNE KRYZY POMIAROWEJ

Streszczenie

Zmiana kierunku strugi przy przepływie przez pojedyncze kolano lub ich układ powoduje asymetryczną deformację pola prędkości. Jeśli dwa kolana umieszczone są w różnych płaszczyznach, może pojawić się również zawirowanie strugi. Wpływ wyżej wymienionych czynników na dokładność pomiaru strumienia masy płynu za pomocą zwężek określano dotychczas wyłącznie na drodze badań eksperymentalnych. W pracy zastosowano badania teoretyczne. Ruch płynu opisano układem równań Reynoldsa domkniętym *k-ε* modelem turbulencji. Układ równań w wersji trójwymiarowej rozwiązywano dla przypadku przepływu płynu przez rurociąg prostoosiowy oraz układy przepływowe zawierające jedno lub dwa kolana umieszczone w różnych płaszczyznach. Celem badań było oszacowanie wpływu pola prędkości przed kryzą na rozkład ciśnienia na ściance oraz współczynnik przepływu przy różnych odległościach przeszkód miejscowych od kryzy. Obie przeszkody miejscowe powodują znaczną zmianę współczynnika przepływu C, co wpływa na dokładność pomiaru. Układ z kolanem generuje ujemną zmianę współczynnika przepływu: $\Delta C < 0$, natomiast dwa kolana dodatnią: $\Delta C > 0$. Wyniki porównano z dostępnymi danymi eksperymentalnymi i stwierdzono ich dobrą jakościową zgodność.