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ANTIALIASING FILTERS IN POWER QUALITY DIGITAL MEASUREMENT SYSTEMS*

Digital signal processing exists in the systems which perform the processing of power waveforms. Each measurement equipment using digital signal processing and computing its discrete spectrum should be equipped with an antialiasing filter which unfortunately creates errors in magnitude and phase spectrum responses. This paper discusses the spectrum distortion analysis of the power signals caused by antialiasing filter implementation in the analog-digital signal processing system paths. The influence of the aliasing phenomenon and a real type magnitude response of the antialiasing filter on the measurement uncertainty of the power signal amplitude characteristic was specified. The values of the minimal sampling frequencies for the power signal spectrum computations were proposed. Use of these frequencies assures the maximum error level at the specified level (0,1%, 1% and 10%). The research includes low-pass realization of the filter magnitude responses approximated by polynomials: Butterworth, Bessel, Chebyschev and elliptic.

Keywords: Antialiasing, filters, digital signal processing system, power quality

1. INTRODUCTION

The current state of technology makes it possible for electrical and non-electrical quantities to be measured. Analog measuring instruments were substituted by digital systems. These systems should be equipped with antialiasing filters. A special example of these systems are devices which process voltage power signals, compute their frequency spectrum and power quality parameters.

Analog-digital conversion generates so-called aliasing errors. In case of wrong selection of the analog-digital conversion parameters, there is a possibility of signal spectrum deformation by the aliasing phenomenon. The influence of this phenomenon on results of research of the spectrum is limited by applying antialiasing filters before the analog-digital converter (ADC). The filter is characterised by limited steepness of slopes, non-zero width of the transition band and real type of spectrum characteristics (Butterworth, Chebyshev, Bessel, elliptic). Establishing the influence of these parameters on computation accuracy of the measured signal spectrum and proposing the optimal choice of them is a very interesting subject.

2. THE ALIASING PHENOMENON, THE DIGITAL SIGNAL PROCESSING SYSTEMS PATH

The spectrum of a sampled signal is presented by the following Eqs. (1), (2):

$$S_p(f) = \frac{1}{T}\sum_{n=-\infty}^{\infty} S\left(f - \frac{n}{T}\right), \tag{1}$$

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where: T - sampling period, $S(f)$ - spectrum of the continuous signal before the sampling operation, n - natural number.

Equation (1) presents the relation between a continuous-signal magnitude spectrum and a sampled-signal magnitude spectrum. The special feature of the equation is the appearance of duplicated magnitude spectrum (aliases) around the total multiple of the sampling frequency 1/T (Fig. 1b).

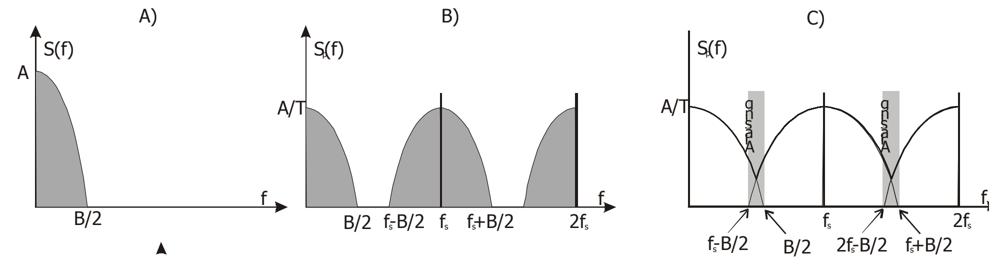

Fig. 1. The principle of creation of aliasing. A) spectrum of continuous signal, B) duplicated spectrum of discrete signal, C) creation of the aliasing phenomenon.

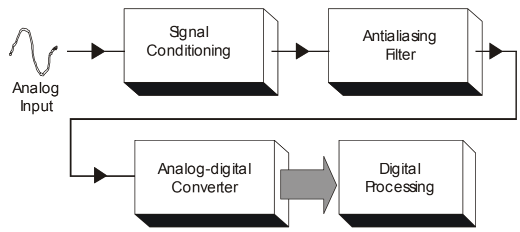

Fig. 2. The structure of a typical analog-digital signal processing path.

A consequence of duplication of the magnitude spectrum is that it is impossible to specify the signal frequency only on the basis of the time samples because one can lead to an infinite number of harmonics through the discrete-time points.

The second phenomenon closely related to spectrum duplication is aliasing. It includes overlapping of duplicated aliases in the frequency domain, when the highest harmonic of a signal is higher than Nyquist frequency (Fig. 1c). Furthermore, it is impossible to perform an inverse Fourier transformation without distorting the analog output signal in the time domain. To avoid this kind of ambiguity, low-pass antialiasing filters are applied between the conditioning circuit and analog-digital converter (Fig. 2) [1,2].

3. THE SAMPLING FREQUENCY AND ANTIALIASING FILTER CUT-OFF FREQUENCY SELECTION CRITERIA IN POWER SIGNALS PROCESSING SYSTEMS

The selection of digital signal processing parameters (especially DFT) starts by specifying the signal bandwidth and the number of considered signal periods (width of time window).

The sampling frequency has to meet the Nyquist condition [1,2] in which the sampling frequency must be at least twice higher than the maximum measured signal frequency. The power signal harmonics controlling systems, which in most cases perform computation of THD factor (Total Harmonic Distortion), should apply a sampling frequency higher than 4 kHz. (THD factor takes into account the first 40 harmonics of the power signal [3]). Thus, the considered minimal power signal bandwidth, which is needed to compute the THD factor, equals 2 kHz. In practice, the sampling frequency is several times higher than 2 kHz. Other systems, which may be sensitive to random short-time duration voltage changes, have to take into consideration a much greater bandwidth (100 kHz). The bandwidth is relevant to the duration of distortion.

The character of the investigated distortions also specifies the number of power signal periods which should be considered in computations. For the sake of averaging features of DFT, the spectrum of the short-time duration distortion should be evaluated, considering the minimal number of periods in contrast to the slow-variable distortion. In such cases, the measuring time-window width should include the maximum number of periods, what simultaneously leads to the increase of the spectrum resolution.

The relation between the spectrum resolution, sampling frequency and number of samples is presented in the following equation:

$$res = \frac{f_s}{N}, \tag{2}$$

where: f_s - sampling frequency, N - number of samples.

To avoid a spectrum leakage phenomenon in DFT computation, the total multiple of signal periods should be taken into account (in case of power signals, the total multiple of 20 ms) [3]. Several ways to eliminate the errors generated by the phenomenon are used in practice.

The first solution is the choice of the time window different from the rectangular shape on the investigated signal [4,5]. In the case of power signals the Hanning window or a rectangular one is used. The application of the rectangular shape window forces synchronization of the sampling frequency with the measured signal frequency (more rigorous than for the Hanning window). Obviously the synchronization should respect the total number of signal periods.

The second solution is the extension of the measuring time window [6,7]. This solution takes into consideration a larger number of power signal periods (i.e. 4-30). This situation results in averaging of short-time duration disturbances on all analysed periods. The accurate location and interpretation of the disturbances may be difficult when such situation occurs. The selection of the sampling frequency depends on the kind of information which is to be obtained as the result of DFT computations. It is significant whether the system should provide data concerning temporary disturbances in power signal or long-term information determining components of the signal spectrum.

The sampling frequency which may be applied in the digital power signal processing systems depending on the number of full periods and number of time samples are shown in Table 1. The table includes only frequencies which are a total multiple of 1 Hz.

Table 1. The sampling frequency (in kHz) used in digital power signal processing systems depending on the number of full periods and time samples.

Amount of samples	AMOUNT OF POWER SIGNAL PERIODS								
N	1	2	4	5	8	10	16	20	25
128	6.4								
256	12.8	6.4							
512	25.6	12.8	6.4	5.12					
1024	51.2	25.6	12.8	10.24	6.4	5.12			
2048	102.4	51.2	25.6	20.48	12.8	10.24	6.4	5.12	4.096
4096	204.8	102.4	51.2	40.96	25.6	20.48	12.8	10.24	8.192

4. THE INPUT SIGNAL MODEL

The voltage power signal is presented by a sinusoidal waveform with a rms value of 230 V and frequency of 50 Hz. Testing of disturbances in the signal, particularly power quality parameters, is simple. The fast and expensive analog-digital converters and wide pass-band conditioning circuits are not required in the systems. There are many types of disturbances in electrical networks beginning with low frequency transients (flickers) to broad-band impulses (electrical discharges, commutation of loads, etc.) in real power distribution networks. The problem of generating the disturbances is described in literature [8,9,10]. The occurrence of impulse-type disturbances in the signal widens its magnitude spectrum. The width of the magnitude spectrum depends on the impulse width, its steepness and amplitude. The increasing number of impulses causes also an increase of the participation of higher harmonics in the signal. In the critical case the signal magnitude spectrum may stay equal in all bands. However, this case occurs very rarely but it allows to check the maximal error level. The input signal model guarantees that the measurement uncertainty will be smaller than computed, even in the most distorted networks.

5. THE MAGNITUDE SPECTRUM MEASUREMENT ERRORS - RESEARCH METHODOLOGY

The measurement uncertainty of the magnitude spectrum in digital measurement systems depends on two factors. The first one is the difference in the pass-band range between shapes of the ideal filter magnitude response (Bode approximation) and real filter magnitude response (approximated by i.e. Butterworth or Bessel polynomials). The second one is the influence of a duplicated spectrum (alias), which was created by the sampling process (aliasing phenomenon). It is necessary to specify the participation of both phenomena in the final uncertainty of the signal magnitude spectrum measurement.

The influence of filter real magnitude response shape.

Filters of real amplitude spectral characteristics are described by some basic parameters:

- type of applied approximation polynomial,
- width of pass-band,
- width of transition-band,
- attenuation in stop-band,
- cut-off frequency,
- filter order.

For the sake of the papers the additional parameter called the width of the utilitarian-band was defined. The parameter defines a part of the pass-band where the error caused by the difference between the ideal and real filter magnitude response is maintained at an assumed level (i.e. 0.1%).

The four most important characteristic types of the filter magnitude responses: Butterworth, Bessel, Chebyshev and elliptic (Cauer) are taken into account in this paper. These characteristics, in most cases, appear in filters included in the digital signal processing systems. The filters are generally available as monolithic circuits realizing one or more types of characteristics . Software using the discussed characteristics types applied in PC computers and in special digital signal processors (DSP) is also popular.

The type of characteristics is closely related to the following parameters:

- pass-band ripple,
- stop-band ripple,
- steepness of the characteristic slopes (related to the width of the transition-band).

Figure 3 presents low-pass realizations of the individual characteristics. The values placed on the frequency axis were normalized in relation to the filter cut-off frequency (fcut).

Each of the characteristics has its special unique properties.

Butterworth filter characteristics. This filter has the flattest possible pass-band magnitude response and moderate steepness of slopes above the cut-off frequency. In the pulse response of the Butterworth filter there occur oscillations rising with the increase of the filter order.

Chebyshev filter characteristics. Filters of this type have steeper attenuation above the cut-off frequency than Butterworth filters. This advantage comes at the cost of the amplitude variation (ripple) in the pass-band. The Chebyshev has even more ringing in its pulse response than Butterworth. The research includes Chebyshev characteristics with the utilitarian-band ripple of $8.69 \cdot 10^{-3}$ dB. This value corresponds to a deviation of the real and ideal characteristics in the utilitarian-band range of about 0.1%.

Bessel filter characteristics. Due to its linear phase response, the filter has excellent pulse response (minimal overshoot and ringing). The magnitude response is not as flat and steep as in the Butterworth filters.

Elliptic (Cauer) filter characteristics. The magnitude response has steeper slopes for a given number of filter order. The phase response of this filter features the worst linearity. The magnitude response in the pass-band and stop-band sections has even ripple. The research includes magnitude response with utilitarian-band ripple equal to $8.69 \cdot 10^{-3}$ dB and 10 dB, 20 dB, 30 dB, 40 dB, 50 dB, 60 dB in stop-band ripple.

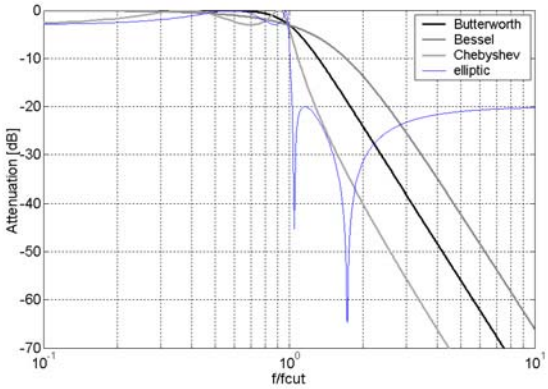

Fig. 3. The filter magnitude responses of the following types: Butterworth, Bessel, Chebyshev and elliptic 20 dB.

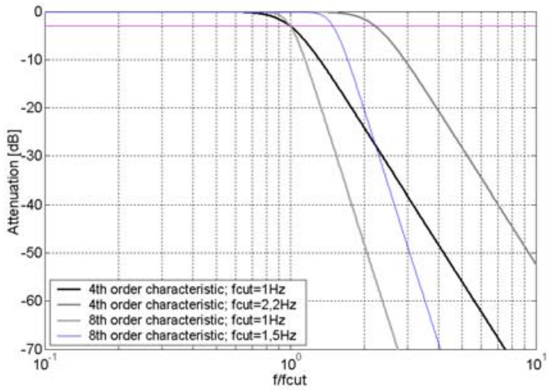

Fig. 4. The Butterworth filter magnitude characteristics of 4th and 8th order before (fcut = 1Hz) and after **cut-off frequency** correction (fcut = 2.2Hz for 4th order and fcut = 1.5Hz for 8th order).

To perform the design of an antialiasing filter and specify the cut-off frequency fcut it is necessary to take into account the fact of 3dB attenuation of the magnitude response. The higher harmonics in an input signal with a bandwidth reaching the cut-off frequency may be attenuated. The error inserted by an antialiasing filter will then equal about 30% for the cut-off frequency. To avoid this distortion it is necessary to correct the filter cut-off frequency. The new value should ensure minimal attenuation in the whole utilitarian-band and keep an error level in the assumed range. For further research, the error level was established as 0.1%. The filters which meet this condition will then not insert a larger error to signal spectrum. The characteristics in Fig. 4 present hypothetical magnitude responses of 4th and 8th Butterworth filter orders before and after cut-off (fcut) frequency correction.

In case of using a 4th-order Butterworth filter, which has to transmit a 1Hz bandwidth signal with a maximum error of 0.1%, a cut-off frequency of about 2.2 Hz should be used. The cut-off frequency increases more then twofold. There is also the influence of the filter order in this relation. For filters of the same type but of the 8th order, the cut-off frequency should be 1.5 Hz. The relation between the cut-off frequency after and before correction may be defined as the cut-off frequency multiplier (CFM).

$$CFM = \frac{f_{cut\ \mathrm{after\ correction}}}{f_{cut\ before\ correction}}\ . \tag{3}$$

The influence of the aliasing phenomenon

The second factor which has the influence on the measurement accuracy of the input signal magnitude spectrum is the aliasing phenomenon. The influence of the phenomenon was determined considering the described input signal model. The magnitude spectrum of the signal is constant in the whole band up to a sampling frequency value.

The principle of estimation of the influence of the aliasing phenomenon consists in specifying the proper sampling frequency. The use of the computed value ensures keeping the error level within the assumed range for the given filter type characteristics, filter order and bandwidth. The principle was presented with the help of a Chebyshev filter of the 4th order, with a bandwidth of 2 kHz and maximum aliasing error equal to 10% (Fig. 5).

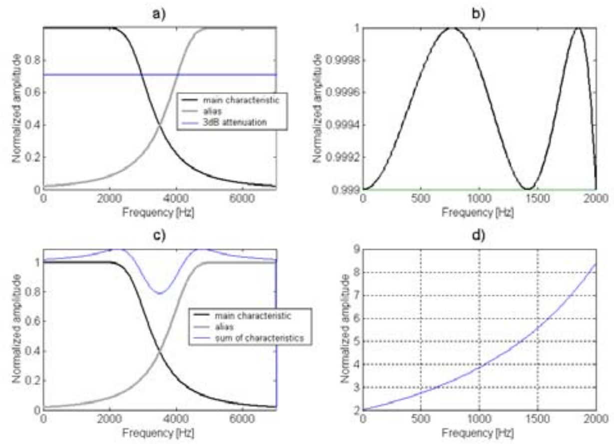

Fig. 5. The principle of performing computation for a 4th-order Chebyshev filter with an utilitarian-band of 2 kHz (fcut = 3 kHz) at assumed maximum aliasing error of 10%. a) comparison of the main band and the duplicated band (alias); b) expanded part of Fig. a; c) comparison of main band, alias and final sum of characteristics; d) the percentage values of errors introduced to the signal magnitude spectrum in the utilitarian-band range (0-2000 Hz) by the aliasing phenomenon.

The previously mentioned input signal is applied to the input of an antialiasing filter. The fact is that the input signal has a constant magnitude spectrum in the whole bandwidth. As a result of filtration we obtain a signal which features the magnitude spectrum exactly fitted to the magnitude response of the filter (Fig. 5a). The cut-off frequency was chosen considering the CFM value (about 3 kHz) in a way to ensure that in the whole bandwidth the maximum difference between the ideal and real magnitude spectrum equals 0.1%.

Figure 5b shows the character of signal amplitude spectrum changes in the range of band not exceeded by value 0.1% (there are also visible pass-band ripples which are characteristic for signals filtered by Chebyshev filters). The grey lines in Fig. 5a and 5c represent a basic amplitude characteristic (alias) duplicated around the sampling frequency. As a result of the sum of these spectra in the pass-band range, an expansion of the signal amplitude spectrum (called the aliasing phenomenon) appears. For the assumed characteristic type, filter order and filter pass-band, the influence of the aliasing phenomenon may be limited by shifting spectra until the error disappears.

The operation results in a specification of the sampling frequency - about 7 kHz. Percentage representation of the error for the sampling frequency was shown in Fig. 5d. The maximum error value in the whole utilitarian-band for this case equals 9%.

6. THE RESULTS OF RESEARCH

The influence of the filter magnitude response shape

The research on the influence of the filter magnitude response shape on the signal spectrum aimed at specifying a correction factor, called the cut-off frequency multiplier (CFM). To obtain a frequency cut-off which guarantees limitation of the error level caused by the differences between the ideal and real filter magnitude response below an assumed value, it is necessary to multiply the signal bandwidth by CFM. The error was established at the 0.1% level as a value which has the minimum influence on the final computation accuracy. The CFM value is a function of the filter magnitude response characteristics type and filter order. The research was performed in order from 1 to 20 and considered the following filter types:

- Butterworth,
- Bessel
- Chebyshev: utilitarian-band ripple 0.0087 dB (0.1%),
- elliptic: utilitarian-band ripple 0.0087 dB (0.1%), stop-band ripple 10 dB, 20 dB, 30 dB, 40 dB, 50 dB and 60 dB.

The CFM values are presented in Table 2 and in Fig. 6, Fig. 7.

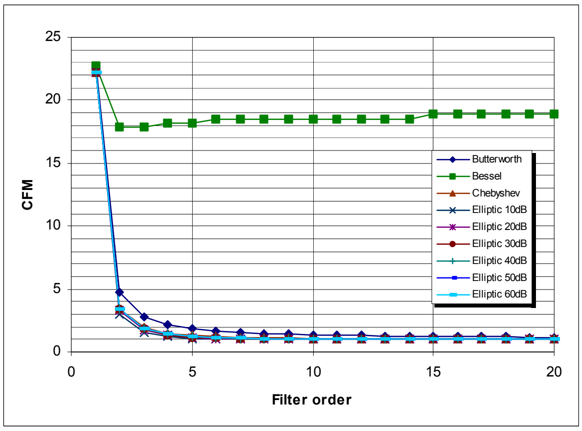

Fig. 6. The CFM dependence on filter order for different characteristics types.

Table 2. Filter cut-off frequency multiplier values (CFM) as a function of the filter order and types of characteristics.

⊢	Filter type

	Butterworth	Bessel	Chebyshev	Elliptic 10dB	Elliptic 20dB	Elliptic 30dB	Elliptic 40Db	Elliptic 50dB	Elliptic 60dB
1	22.73	22.73	22.22	22.22	22.22	22.22	22.22	22.22	22.22
2	4.74	17.86	3.41	2.99	3.28	3.37	3.40	3.41	3.42
3	2.82	17.86	1.92	1.56	1.73	1.82	1.88	1.89	1.91
4	2.18	18.18	1.49	1.20	1.30	1.37	1.42	1.45	1.47
5	1.86	18.18	1.31	1.08	1.13	1.18	1.22	1.25	1.27
6	1.68	18.52	1.22	1.03	1.06	1.09	1.12	1.15	1.17
7	1.56	18.52	1.15	1.01	1.03	1.05	1.07	1.09	1.10
8	1.47	18.52	1.12	1.01	1.02	1.03	1.04	1.06	1.07
9	1.41	18.52	1.09	1.00	1.01	1.02	1.02	1.03	1.04
10	1.37	18.52	1.07	1.00	1.00	1.01	1.01	1.02	1.03
11	1.33	18.52	1.06	1.00	1.00	1.01	1.01	1.01	1.02
12	1.30	18.52	1.05	1.00	1.00	1.00	1.01	1.01	1.01
13	1.27	18.52	1.05	1.00	1.00	1.00	1.00	1.01	1.01
14	1.25	18.52	1.04	1.00	1.00	1.00	1.00	1.00	1.01
15	1.23	18.87	1.03	1.00	1.00	1.00	1.00	1.00	1.00
16	1.21	18.87	1.03	1.00	1.01	1.00	1.00	1.00	1.00
17	1.20	18.87	1.03	1.00	1.00	1.00	1.00	1.00	1.00
18	1.19	18.87	1.02	1.00	1.00	1.01	1.00	1.00	1.00
19	1.18	18.87	1.02	0.99	1.00	1.00	1.01	1.00	1.00
20	1.17	18.87	1.02	0.99	1.02	1.00	1.00	1.00	1.00

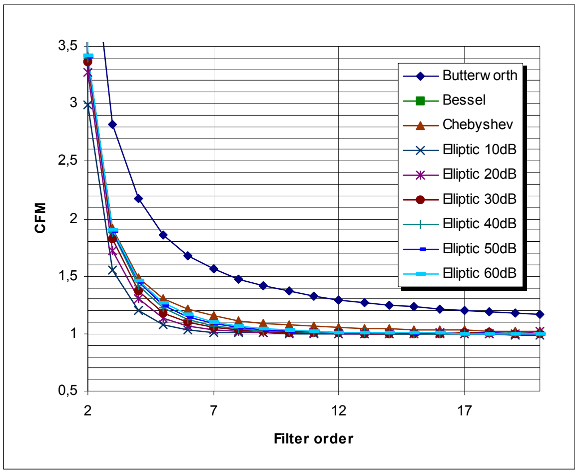

Fig. 7. The CFM dependence on filter order for different characteristics types (the expanded part of Fig. 6).

The characteristics shown above present a decrease of CFM with an increase of the filter order until about the 6th order. A further increase of the filter order does not cause a significant decrease of CFM and stays saturated at the 1 value. An exception is the Bessel filter, where for orders 2 to 20, CFM varies between values 18 and 19. As a consequence, it is necessary to use wide-band filters or a narrow-band, but lower accurate characteristics filter in the band range (1% or more). The narrow-band filtering circuits which implement this type of magnitude response, will insert relatively high distortion for higher harmonics. Thus, these circuits find application in systems where maximum linear phase response is required and accuracy of magnitude characteristic is not so significant.

The elliptic and Chebyshev filters reach a CFM factor which equals 1, the fastest. The filters show stronger non-linearity of the phase response and consequently ripples occur in the time domain of the output signal. A compromise between linear phase response and relatively low CFM value is the Butterworth filter.

The influence of the aliasing phenomenon.

The aim of the performed research is to specify a minimum sampling frequency which should be applied in the system to ensure the maximum aliasing phenomenon error below an assumed value. The frequency is dependent on the filter order and utilitarian-band width. The computations concern cut-off frequency correction with the help of the CFM factor. Thus, all the characteristics insert an error of a maximum value of 0.1% to the filter pass-band.

The computation was performed for the following parameters:

- error values: 0.1%, 1% and 10%;
- width of utilitarian-band: 2 kHz, 2.5 kHz, 10 kHz, 25 kHz and 50 kHz;
- filter orders: 1-20
- types of filter magnitude responses:
- Butterworth,
- Bessel,
- Chebyshev - utilitarian-band ripple value 0.0087 dB (0.1%),
- Elliptic - utilitarian-band ripple value 0.0087 dB (0.1%) and stop-band ripple value: 30 dB, 40 dB, 50 dB and 60 dB.

The utilitarian-band values were selected to make the implementation in the systems of measuring slow- and fast-variable power signals possible. The 2 kHz and 2.5 kHz utilitarian-bands are used to measure the THD factor in the power signal, which requires considering the first 40 harmonics [3].

The measurements of the spectrum content are very often made up to the 50th harmonic, inclusive. Thus, the research considers a 2.5 kHz utilitarian-band. The remaining ranges of bands may be used to measure the wide-band signals or disturbances like over-voltages.

The analysis of errors introduced by the aliasing phenomenon includes all filter types considered during the CFM factor testing, with the exception of 10 dB and 20 dB elliptic filters. According to the definition, filters of this type do not have large stop-band attenuation and can insert significant errors to the magnitude characteristic of signals.

Figures 9 to 14 show the results of analyses. Figure 8 shows the manner of data representation in the diagrams.

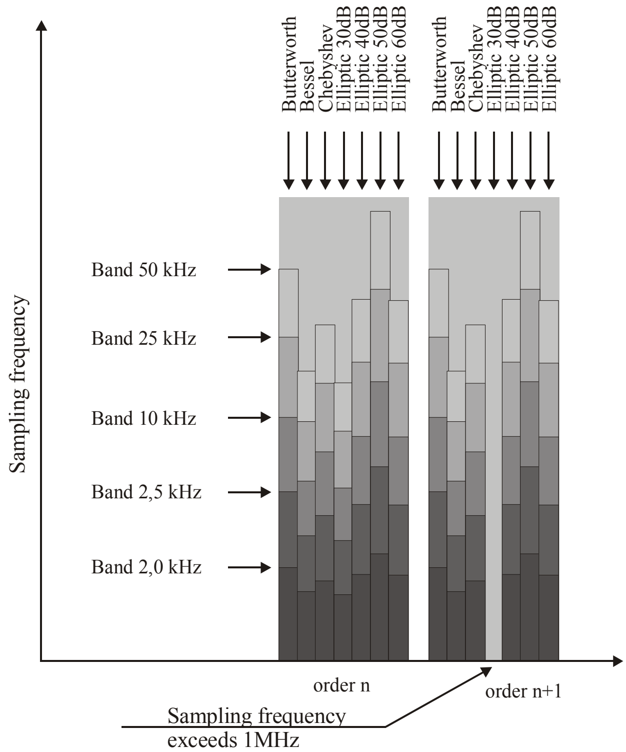

Fig. 8. The manner of data representation in the diagrams.

Each of the filter orders is represented by a group of computations for 7 characteristic types. Proper columns in the groups were assigned to the individual characteristic types. The first column shows data concerning the Butterworth filter, the second a Bessel filter, the third a Chebyshev filter, elliptic 30 dB, 40 dB, 50 dB filters and the final one presents data proper for the elliptic 60 dB filter. The columns are divided into 5 parts which correspond to proper filter pass-bands. Lack of a column for the given filter order means that the sampling frequency exceeds 1 MHz or cannot be specified (i.e. in case when the filter characteristic is unable to ensure an assumed attenuation).

Sampling frequencies marked from diagrams visibly decrease with an increase of the filter order.

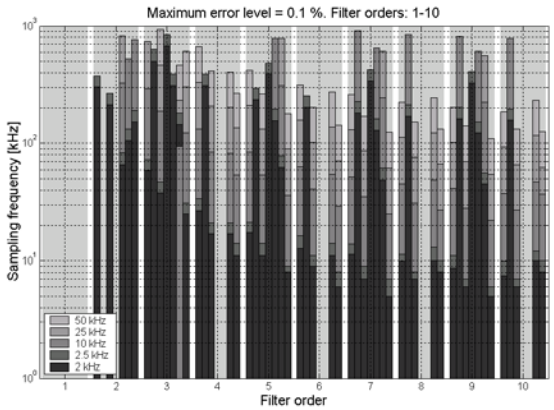

Fig. 9 The dependence of the sampling frequency selection on filter order and utilitarian-band for maximum errors introduced by the aliasing phenomenon equal to 0.1% (orders 1-10).

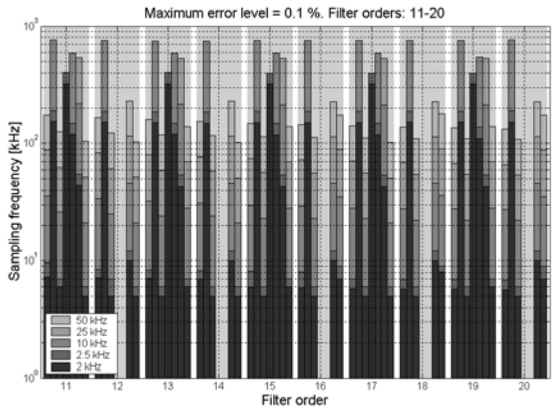

Fig. 10 The dependence of the sampling frequency selection on filter order and utilitarian-band for maximum errors introduced by the aliasing phenomenon equal to 0.1% (orders 11-20).

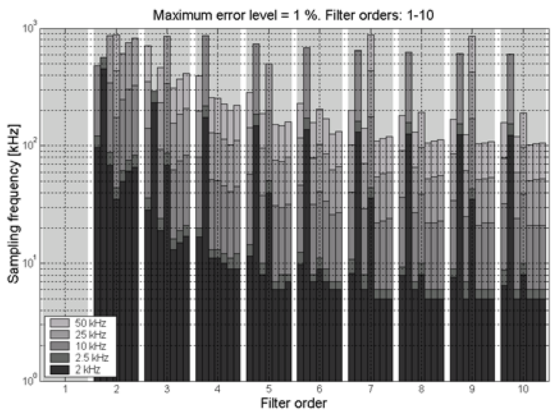

Fig. 11 The dependence of the sampling frequency selection on filter order and utilitarian-band for maximum errors introduced by the aliasing phenomenon equal to 1% (orders 1-10).

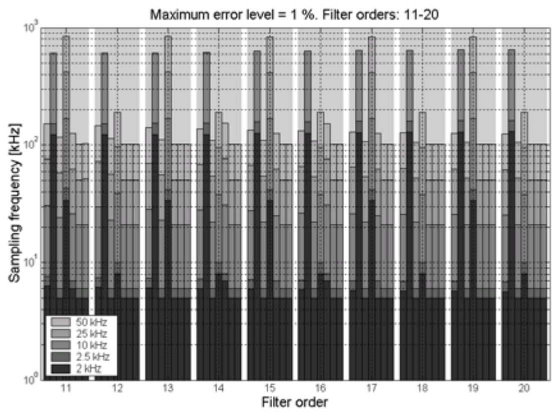

Fig. 12 The dependence of the sampling frequency selection on filter order and utilitarian-band for maximum errors introduced by the aliasing phenomenon equal to 1% (orders 11-20).

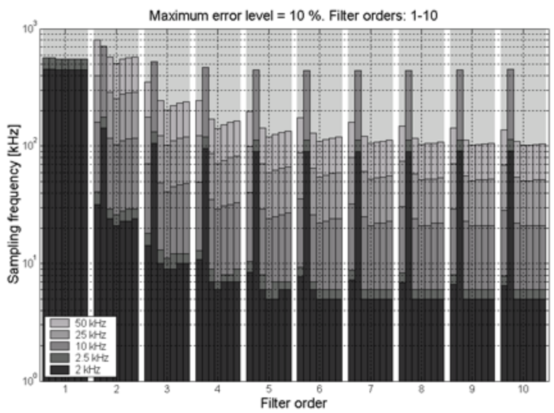

Fig. 13 The dependence of the sampling frequency selection on filter order and utilitarian-band for maximum errors introduced by the aliasing phenomenon equal to 10% (orders 1-10).

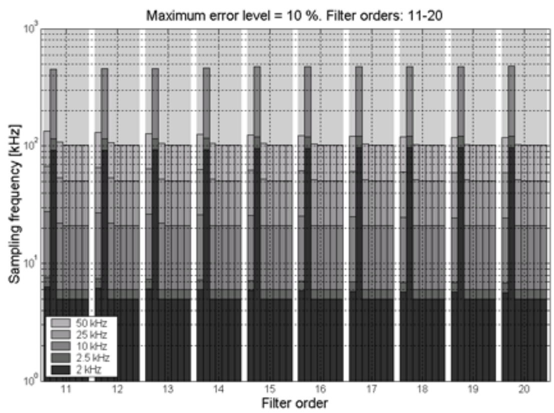

Fig. 14 The dependence of the sampling frequency selection on filter order and utilitarian-band for maximum errors introduced by the aliasing phenomenon equal to 10% (orders 11-20).

This decrease is visible in the diagrams assigned for the error levels of 1% and 10%. This trend maintains until approximately the 6th order. Above this value the sampling frequency changes for each band are minimal. The use of filters higher than 7th order seems to be groundless.

The remaining elliptic filters (30 dB and 40 dB) do not have such exactly monotonically decreasing character. Sampling frequencies which should be applied in the system are alternately this higher and lower and this can be seen in diagrams assigned to the error levels of 0.1% and 1%. It is closely related to the stop-band elliptic magnitude response shape, where a ripple characteristic exists. The ripples have special effect at low attenuation levels in stop-bands, which can be particularly seen in the elliptic 30 dB filter. According to the definition, these filters have low stop-band attenuation. Thus, there are no specifc sampling frequencies for the error level of 0.1% in the diagrams (lack of appropriate columns).

On the average, the highest sampling frequencies are obtained for the Bessel filter (even above 400 kHz for 1% error), and are the result of mild transition from a pass-band to a stop-band and the high CFM values. The elliptic 60 dB filter presents the lowest sampling frequencies, but from the 8th order it has values comparable to Butterworth and Chebyshev filters.

7. CONCLUSION

The paper presents the estimation of the influence of errors inserted to the magnitude spectrum of the transmitted signal by an antialiasing filter. The errors are generated by filter amplitude response approximation with the help of real characteristic (Butterworth, Chebyshev, Bessel, elliptic) and creation of the aliasing phenomeneon after performing a discretization process of the input analog signal.

The impact of the first type of error may be reduced by using a correction factor (CFM). The filter utilitarian-band width should be multiplied by the CFM factor. The resulting value is a filter cut-off frequency which ensures the maintenance of error at the level below 0.1%.

The second error, after considering the CFM factor, was established at levels of 0.1%, 1% and 10%. For such values, the minimal sampling frequencies were established and they should be applied to the system as a function of the filter order, characteristic type and width of the filter utilitarian-band.

Assuming the width of the filter utilitarian-band, aliasing phenomenon error and the dynamics of signal changes (represented by the filter order), a minimal sampling frequency is obtained while designing the system, as a result of computations. The sampling frequency values should consider synchronization with the input signal.

A system designed for transmitting fast variable signals should apply a low-order antialiasing filter with a maximally linear phase response. The Bessel filter has such characteristics. To minimize nonlinearity of the amplitude characteristics in the pass-band in filters of this type, the cut-off frequency must be relatively high. A better solution is the use of a Chebyshev filter (with small pass-band ripple), which does not require such a high cut-off frequency (on the average 6 to 7 times lower than for a Bessel filter). The phase response of the Chebyshev filter is more nonlinear than in the case of a Bessel filter. A compromise solution is the use of low-order Butterworth filters having better pass-band magnitude response, linearity and phase response nonlinearity than the Chebyshev filter. The sampling frequency of the system with the application of a 4th-order Butterworth filter and 10 kHz pass-band equals 80kHz. The cut-off frequency is then 22 kHz.

In applications measuring low-variation signals, practically all filter types (i.e. THD factor measurement) may be used. The best results can be obtained with the elliptic 60 dB filter. To obtain a magnitude response measurement error at the 0.1% level and to perform measurement in the range of 2 kHz, a 7th-order filter may be used. The filter cut-off frequency should be 2.2 kHz, at the minimal sampling frequency of 5 kHz. Finally, considering frequency synchronization, the sampling frequency may be chosen at 5.12 kHz or 6.4 kHz.

The computations specify the maximum error level which may appear in the input signal magnitude spectrum, after filtration by a real magnitude response filter. Errors have a boundary character and are computed assuming the worst case condition - that is where accumulation of individual distorting phenomena and the input signal magnitude spectrum are equal in the whole band. The results of computation may give satisfying effects in the strongly- as well as lightly-distorted environments.

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FILTRY ANTYALIASINGOWE W CYFROWYCH SYSTEMACH POMIARÓW JAKOŚCI ENERGII ELEKTRYCZNEJ

Streszczenie

Artykuł przedstawia oszacowanie wpływu błędów wnoszonych przez filtr antyaliasingowy do charakterystyki amplitudowej przenoszonego sygnału w systemach cyfrowego przetwarzania sygnałów elektroenergetycznych, a w szczególności systemów pomiarowych parametrów jakości energii elektrycznej. Błędy te generowane są w wyniku aproksymacji charakterystyki amplitudowej filtru za pomocą charakterystyki rzeczywistej (Butterwortha, Czebyszewa, Bessla, eliptycznej) oraz powstawania zjawiska aliasingu po przeprowadzeniu procesu dyskretyzacji wejściowego sygnału analogowego.

Zaproponowano minimalne częstotliwości próbkowania dla zastosowań obliczania widma sygnałów elelktroenergetycznych zawierających zakłócenia wolno- i szybkozmienne. Stosowanie wyznaczonych wartości zapewnia utrzymanie poziomu błędu na określonym poziomie (0.1%, 1% oraz 10%).

Badane błędy mają charakter graniczny. Są one wyznaczone przy założeniu najgorszego przypadku, tj. gdy występuje sumowanie się błędów generowanych przez poszczególne zjawiska zniekształcające oraz przy założeniu stałości widma amplitudowego sygnału wejściowego w całej szerokości pasma. Wyniki obliczeń mogą zatem z powodzeniem dawać zadowalające efekty zarówno w środowiskach słabo jak i silnie zakłóconych.