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ANGLE MEASUREMENT OF RIGHT ANGLE PRISMS USING OPTICAL COORDINATE MEASURING INSTRUMENT

A new technique for calibrating the angles of right angle prisms using an optical coordinate measuring instrument is presented in this work. The technique is an optical non-contact method that is based on focusing of images of the measured elements. Measurements of the angles of optical components are very important since they affect the accuracy and precession of the measurement techniques. The results and their uncertainties show the validity of the method.

Keywords: Right angle prism, angle measurement, optical coordinate measuring instrument

1. INTRODUCTION

Right angle prisms are essential in optical systems and instruments. The calibrations of the angles of such prisms are very important since their errors affect the accuracy of the measurements realized by the optical systems. For example, electronic distance meters use corner cube right angle prisms or retroreflectors for reflecting the optical rays. The errors of these prisms or retroreflectors cause the optical beams to be deviated, which produces errors in the measurements of the instruments. The right angle prism can be used to deviate rays normal to the incident face by 90° and to give the mirror erected and inverted image as shown in Fig.1-a [1].

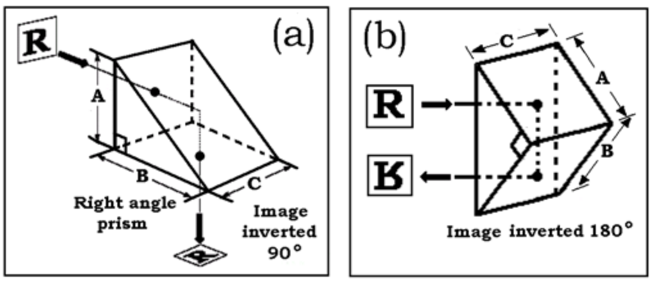


Fig.1. The right angle prisms of dimensions A, B and C are used for; a) the rays are deviated by 90° to give a mirror inverted image, b) the image is reflected and inverted two times and the beam is deviated by 180°.

The second application known as Porro prism or retroreflector is the same prism but it is used in a different orientation where the image is reflected and inverted two times and the beam is deviated by 180° [1]. Measurements of the angles can be realized by various methods that are mainly mechanical and optical. The mechanical methods depend on using mechanical touch devices such as master angle gauges, indexing tables and goniometers, however, these methods suffer from limitations of the accuracy [2, 3, 4]. The optical methods are more accurate due to the application of the wavelength of light as the measuring unit in the optical instruments such as spectrometers, autocollimators, spectrographs [5-8] and laser interferometers [9]. However, the optical methods have disadvantages of high costs and are time-consuming; also the measurements which are carried out in some points without any idea about the results if the position on the surface of the prism face where the angle is measured is changed. Thus the shape of the prism's surface affects the accuracy of the measured angles if it is not ideally flat as shown in Fig. 2. [9].

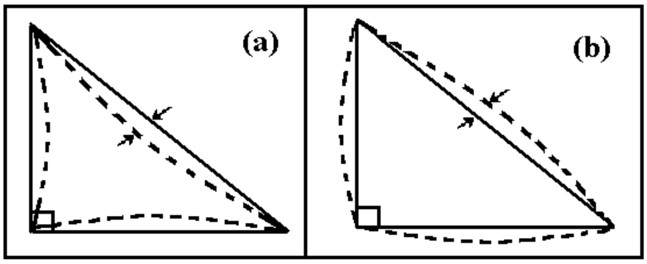


Fig.2. A right angle prism with deviated non flat (exaggerated) faces, the prism's angles do not sum up to 180°; a) convex faces, b) concave faces.

The angles of a right angle prism measured by optical methods will be varied depending on the place of the points on the surface of the convex faces (Fig. 2a) or as shown in Fig. 2b in case of concave faces.

2. THEORY

The optical coordinate measuring Instrument (OCMI) is a computerized optical measuring system that enables to determine the dimensions of spatially shaped elements without contact. Because of this useful feature hard and soft elements can be measured similarly. Measurements of dimensions and angles of optical components are very important to assess their accuracy and precession that are needed to comply with specifications. In this work the angles of four right angle prisms are measured, applying OCMI to assess their accuracies.

The OCMI applied in this work is a Computer Numerical Controlled (CNC) instrument produced by Werth Messtechnik Gmbh. It belongs to the German Institute of Micro-technology. The instrument is placed in the clean room's area in very good environmental conditions. The temperature equals (20 ± 0.5)°C and humidity equals 60%. Also, the instrument software has the possibility to compensate environmental conditions provided by the user. The accuracy of the OCMI instrument is enhanced by applying special software error correction that compensates and corrects the errors resulting from the mechanical and optical parts of the OCMI instrument such as length, position, rotation, straightness, squareness inaccuracies and optical focusing. The data of these inaccuracies are obtained from the periodical calibration of the instrument and it is corrected during the measurements. The OCMI uses an optical system that consists of an external cooled white light source, a fiber optical cable to provide white light to the magnification lenses, a CCD high resolution camera and image grabber software and hardware. The OCMI measures the object's dimensions applying geometrical elements such as point, line, plane, area, circle, cylinder, ball and cone by probing a number of points located on the surface of the measured element. The element dimension and location on the OCMI coordinate system are determined by using the values of the point coordinates. The OCMI probes the points applying the optical focusing method either with the aid of the measuring person or by auto focus technique. The location of a point (P) of coordinates (x, y, z) on the surface of an element can be determined by OCMI as the position of the optical head plus focal length of the applied lens attached vertically in z axis of the OCMI head, therefore:

$$P(x, y, z, f) = P(x), P(y), [P(z) + P(f)], \tag{1}$$

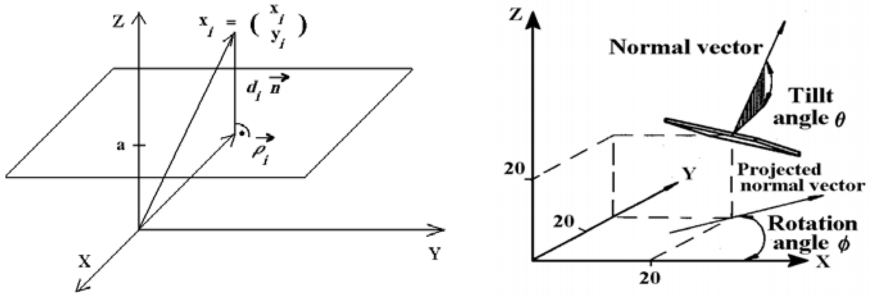
where: $P(x)$, $P(y)$, $P(z)$ are the positions of the head of the OCMI due to x, y, z axes, respectively. $P(f)$ is the position of the focused point on the surface of the element which is detected by the optical head of the OCMI and f is the focal length of the applied magnifying lens.

The position of the plane can be determined by approximation of the measured points by the method of least squares. The plane (Hemdt method) is determined similar to the case of a straight line. The plane in Cartesian coordinates can be represented by a vector formula as follows [10]:

$$\vec{n} \bullet \vec{x} - a = 0, \tag{2}$$

where: $\vec{n}$ - normal vector of the plane, as $|\vec{n}|$, $\vec{x}$ - vector driving the points of the plane, a - distance from origin point of Cartesian coordinates.

The plane in Cartesian coordinates (Fig. 3a) can be determined by measuring at least four points which are distributed near the four angles of the plane.



a) b)

Fig. 3. Position of the plane in Cartesian coordinates, a) a theoretical illustration of the plane [10], b) an experimental description of a plane on the measuring software of the instrument [11].

The plane is described in the measuring software of the instrument by means of the center of mass and the spatial position of the normal vector. The normal vector is always perpendicular to the plane at the center of mass of the plane. The angle of rotation of the plane's normal vector (ϕ) is derived from the positive x-axis and the normal vector projected into the x/y plane. The angle of tilt (θ) describes the position of the normal vector in relation to the x/y plane [11].

3. METHOD

The right angle prism is placed on the middle of the OCMI glass table so that its base faces its apex angle (90°) as shown in Fig. 4. The right side of the measured prism represented in Fig. 5 by plane 1 is measured as a plane, applying the OCMI. The plane is measured by localizing (probing) at least 6 points (each with three coordinates in x, y and z) on the surface of the plane. The localized points are distributed near the corners of the plane. The points are localized (probed) by the optical sensor head which is equipped with magnification lenses and the optical imaging technique. In the present work a magnification lens power of 20 x is used for localizing and probing the focused points [11].

The optical imaging technique consists of a high resolution video camera, a powerful and fast frame grabber, a high resolution color video graphics display and an optical image processor with image recognition software. The image processor is a non-contact optical sensor which generates contours from the video images. It supplies the measuring software with the form elements. The magnification lens forms an image of the object to be measured on the CCD-chip of the Video camera. The chip consists of a matrix of photo diodes that are light sensitive pixels. Each pixel, depending on the intensity of light incident on it, has a specific electrical potential which is read out as a video signal and transmitted to the image processor.

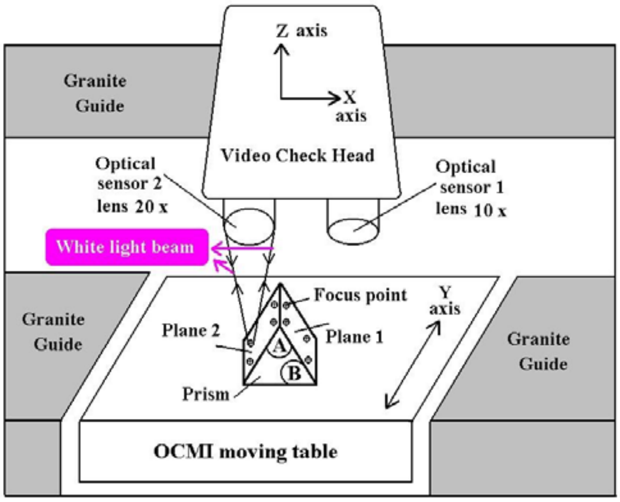


Fig. 4. Measurement of base and apex angles of right angle prism using an OCMI instrument.

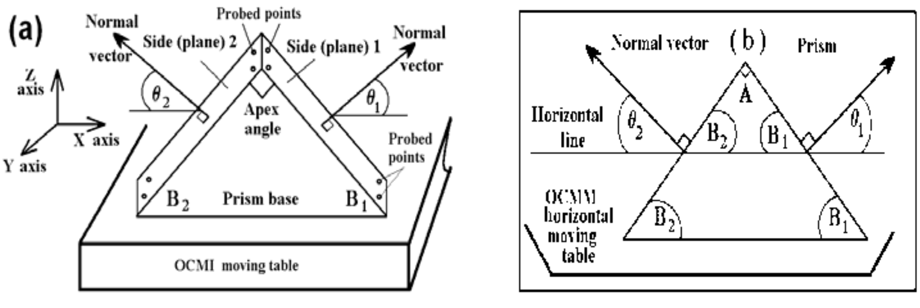


Fig. 5. Determination of the angles of right angle prism using OCMI measuring software; a) 3-D illustration of measurements of planes and points of the prism, b) geometrical description of the prism angles.

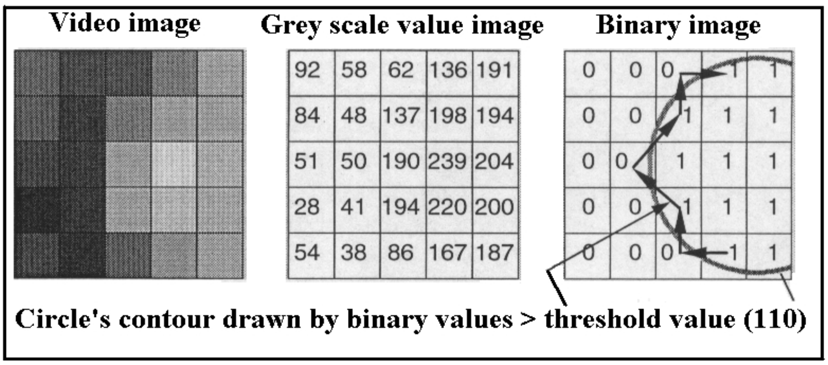


Fig. 6. The Video image is converted to a gray scale value image, then it is converted to binary image. The threshold value 110 produces a contour of the form element, for example a circle.

At the image processor, a gray scale value from 0 to 255 within the image processor matrix is assigned to each pixel in accordance with its voltage potential. The frequency distribution of gray scale values (histogram) allows for determining the average threshold value. All gray scale values above the threshold value are evaluated as 1 (white), and all values below are evaluated as 0 (black). The boundaries between 0 and 1 draw a contour of the measured point. Pixels forming the contour are linked together by means of a logical algorithm; they are processed as shown in Fig. 6. The user chooses in the measuring software options which element should be assigned to the generated contour. The image from the video camera is displayed on the monitor and can be evaluated. The localization process of points is realized by focusing the lens of the head of OCMI on the surface of the right angle prism by moving it up and down until a bright sharp image is obtained in the monitor of the optical imaging system, then the point is measured and its coordinates are recorded.

The OCMI software determines the position of the plane and its normal vector by applying measured point coordinates and equations of the plane as shown in Fig. 5a. Since the measured plane 1, which is the right angle prism's side 1, is inclined by the prism base angle B1 (Fig. 5a), therefore, its normal vector is inclined from the horizontal line of the OCMI moving table by: an angle θ_1 as shown in Fig. 5b. From the geometry of figure, the right angle prism base's angle is given by:

$$\mathrm{B1} = \theta_1. \tag{3}$$

The results of plane 1 are stored in the OCMI computer memory. The same measuring process is repeated for the right angle prism's side 2 and similarly the second right angle prism base angle B2 is determined from the equation:

$$\mathrm{B2} = \theta_2. \tag{4}$$

The results of plane 2 and its normal vector are stored. The Apex angle (A) of the right angle prism which is the angle between two right angle prism sides 1 and 2 (plane 1 and 2) are determined using stored equations of the two planes. The two planes are determined using coordinates of the probed points. Experimentally, the angle between the normal vectors of the two planes 1 and 2 is determined by creating a new element option of the OCMI software using the measured elements of plane 1 and 2 and their normal vectors.

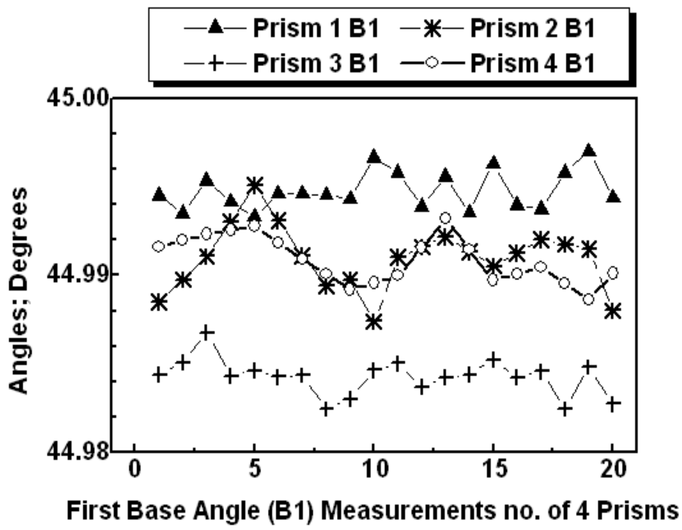
4. RESULTS

Measurements of the base and apex angles of the four right angle prisms are repeated several times to study the accuracy of measurements. The measurement procedure is programmed by the OCMI instrument's computer applying the learning facility. Then the measuring program is repeated twenty times automatically and the results are saved. Results of first base angles B1 of the four right angle prisms are plotted in Fig.7a. Results of the angles B1 are distributed on maximum and minimum values around the mean of results due to the effect of random errors. The four prisms are different in values of the base and apex angles because of the differences resulting from the errors of form and surface shape of the four measured prisms. For instance, the lowest error in B1 angle is prism no. 1, while the largest error is prism no. 3 and the other two prisms are intermediate in their errors of B1 angles. Results of the angle B1 of prism P1 are varying from 44.9933° (44°59’35.88”) to 44.997° (44°59’49.2”), the mean of results is 44.99477° (44°59’41.17”) with a standard uncertainty which is the standard deviation of the mean equals to ±0.00109° (±3.9’’) .

Results of second base angle B2 of prism no. 1 (Fig. 7b) are varying from 44.98276° (44°58’57.9”) to 44.98828° (44°59’17.8”), the mean of results is 44.98582° (44°59’9.0”) with a standard uncertainty of the mean equal to ±0.0013° (±4.7”).

Results of Apex angle (A) of prism no.1 (Fig.8) are varying from 89.96956° (89°58’10.4”) to 89.98053° (89°58’49.9”), the mean of results is 89.97636° (89°58’34.9”) with a standard uncertainty of the mean equal to ±0.00278° (±10”).

a)



b)

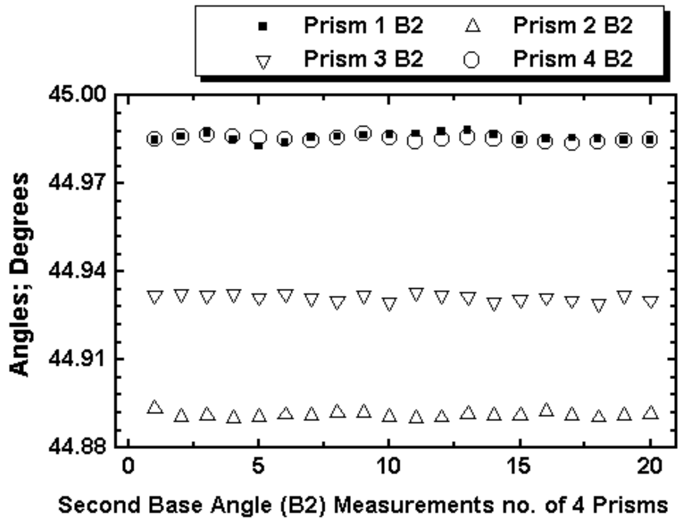


Fig. 7. The spreading of results of base angles of the right angle prisms; a) results of 1st base angles (B1), b) results of 2nd base angles (B2).

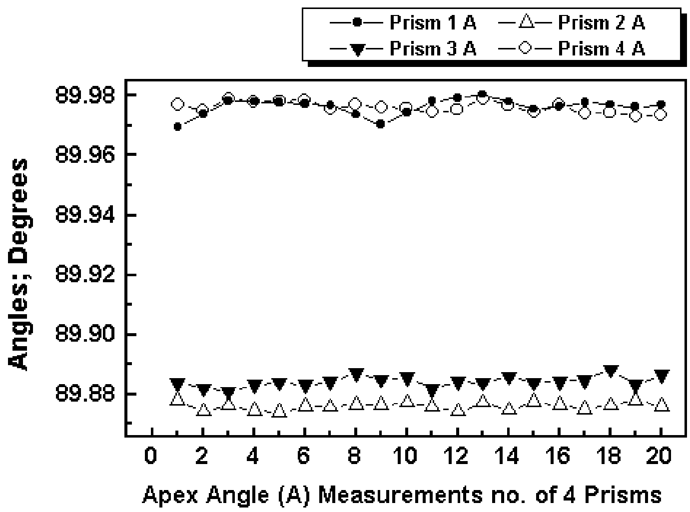


Fig. 8. The spreading of results of apex angles (A) for the right angle prisms.

The deviation of measured angles from the designed value can be identified as angle errors of apex and base angles of the four right angle prisms. These errors are plotted together in a column type graph to demonstrate the relations among the three angles of each prism as shown in Fig. 9.

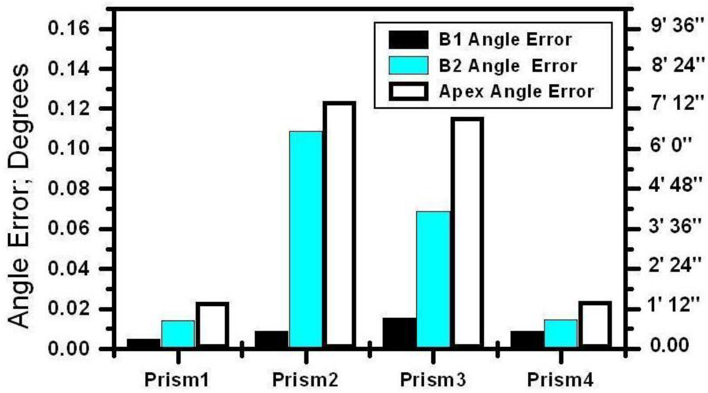


Fig. 9. The angle errors of apex and base angles of the four right angle prisms are determined as the deviation of the measured value from the designed one.

The figure shows that the differences between the errors of apex and base angles of the 1st prism and 4th prism are small in comparison with the angles of 2nd and 3rd prisms. This fact clarifies that these errors are resulting from defects in the measured prisms since if these errors were created by the measuring OCMI instrument then the differences between the apex and base angles of each prism would be comparable to each other.

The results of measurements of base and apex angles for the four right angle prisms with their mean and uncertainty evaluated by the standard deviation of the mean are given in Table 1.

Table 1. Results of the base and apex angles for the four right angle prisms.

Prism-angle	Minimum Degrees	Maximum degrees	Meas. error = Max.- Min.	Mean degrees	Angle error of_designed value	Uncert. (Standard deviation) u
P1- B1	44.9933°	44.997°	0.0037=13.3	44.99477°	0.00523°=18.83’’	0.00109° = 3.9’’
P2- B1	44.98739°	44.99506°	0.00767=27.6	44.99097°	0.00903°=32.51’’	0.00181° = 6.5”
P3- B1	44.98246°	44.98676°	0.0043=15.5	44.98427°	0.01573°=56.6’’	0.00102° = 3.7 ”
P4- B1	44.98861°	44.99318°	0.00457=16.5	44.99087°	0.00913°=32.87”	0.00132° = 4.8 ”
P1- B2	44.98276°	44.98828°	0.00552=19.9	44.98582°	0.01418°=51.05’’	0.0013° = 4.7 ”
P2- B2	44.89°	44.89335°	0.00335=12.1	44.89116°	0.10884°=6'31.8’’	0.000806° = 2.9”
P3- B2	44.92893°	44.93291°	0.00398=14.3	44.93118°	0.06882°=4’7.75’’	0.00113° = 4.1 ”
P4- B2	44.9837°	44.98692°	0.00322=11.6	44.98522°	0.01478°=53.21’’	0.00083° = 3 ”
P1- A	89.96956°	89.98053°	0.01097=39.5	89.97636°	0.02364°=1’25.1’’	0.00278° = 10 ”
P2- A	89.87399°	89.87797°	0.00398=14.3	89.87592°	0.12408°=7’26.7’’	0.00121° = 4.4 ”
P3- A	89.88103°	89.88829°	0.00726=26.2	89.88423°	0.11577°=6’56.8’’	0.00181° = 6.5 ”
P4- A	89.97326°	89.97915°	0.00589=21.2	89.97616°	0.02384°=1’25.8’’	0.0018° = 6.5 ”

The results of measurements of base and apex angles of the four right angle prisms show possibility of the method for measurements and checking of the prism angles and optical components with an accuracy of a few seconds.

The information obtained from the results of prism angle measurements shows the degree of the accuracy that is possibly affected by some sources of errors. In order to study the errors to increase the accuracy of angles measured by this technique, the distribution of results must be studied and fitted to a known statistical distribution. As an example, results of B1 prism base angles of the four prisms are examined by various distributions using an IBM PC. It is found that the B1 results are best fitted to the normal distribution as shown in Fig. 10.

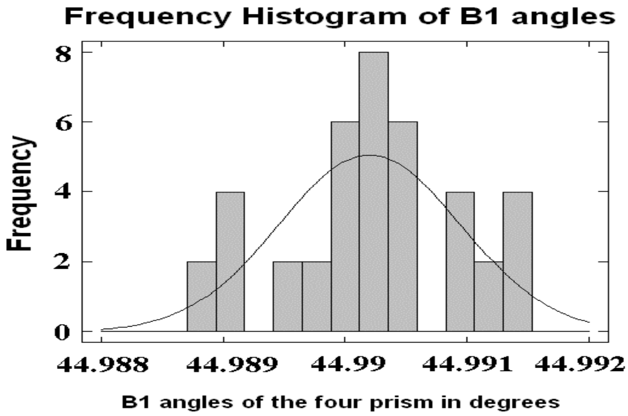


Fig. 10. The frequency histogram of B1 angles of 4 prisms that are fitted to normal distribution.

The vertical gray bars in the figure represent the frequency of the fitted B1 angles and the curve is the normal distribution curve. The fitting of results is investigated by Chi-square and Kolmogrov-Smirnov [12] one sample tests. The Chi-square test equals 4.761 and has significant level of fitting (α = 0.0924) which is greater than 0.05, at a confidence interval of 95%. The Kolmogrov-Smirnov test shows that the estimated overall statistic DN = 0.098213 and the approximate significance level (α = 0.999989) which is greater than 0.05 (Conf. interval 95%). Therefore, the results are best fitted to normal distribution since the significance level of fitting is greater than 0.05 (Conf. level; 95%).

The accuracy of prism angle measurements can be improved by introducing a special error correction program to the measuring instrument software. For example, such correction is realized by the polynomial regression method. The method fits a model relating the dependent variable to the independent variable by using the least squares method for fitting a curve to the examined results, as shown in Fig. 11a. The model used in the polynomial regression (Table 2) is of the 5th order and is given as follows:

$$Y = A + B1\ X + B2\ X2 + B3\ X3 + B4\ X4 + B5\ X5. \tag{5}$$

Table 2. The method of fitting a curve to the results of apex angles of the 1st prism.

Polynomial Regression for Prism 1 Apex Angles: $Y = 89.95622+0.01643\ X - 0.00426\ X^2 + 4.67616 \times 10^{-4}\ X^3 - 2.27352\times10^{-5}\ X^4 + 4.04095\times10^{-7}X^5$				
Parameter	Value	Error	t-Value	Prob>|t|
A	89.95622	0.00441	20383.63009	<0.0001
B1	0.01643	0.00387	4.24471	8.16476E-4
B2	-0.00426	0.00108	-3.96142	0.00142
B3	4.67616E-4	1.26208E-4	3.70512	0.00235
B4	-2.27352E-5	6.55973E-6	-3.46587	0.00378
B5	4.04095E-7	1.24414E-7	3.24799	0.00584
R-Square(COD)	SD	N	P	
0.62374	0.00199	20	0.01046	

where: parameter A is the intercept value and its standard error, B1, B2, B3, B4, B5 parameters are the slope values and their standard errors estimated by the model. The t-statistical values are values for testing if the parameter equals zero, where t equals the parameter estimate/standard error of the estimate. The p-value is the probability that R-square is zero for the corresponding t-values. The R-square is the coefficient of determination. N is the number of data points. SD is standard deviation of the fit.

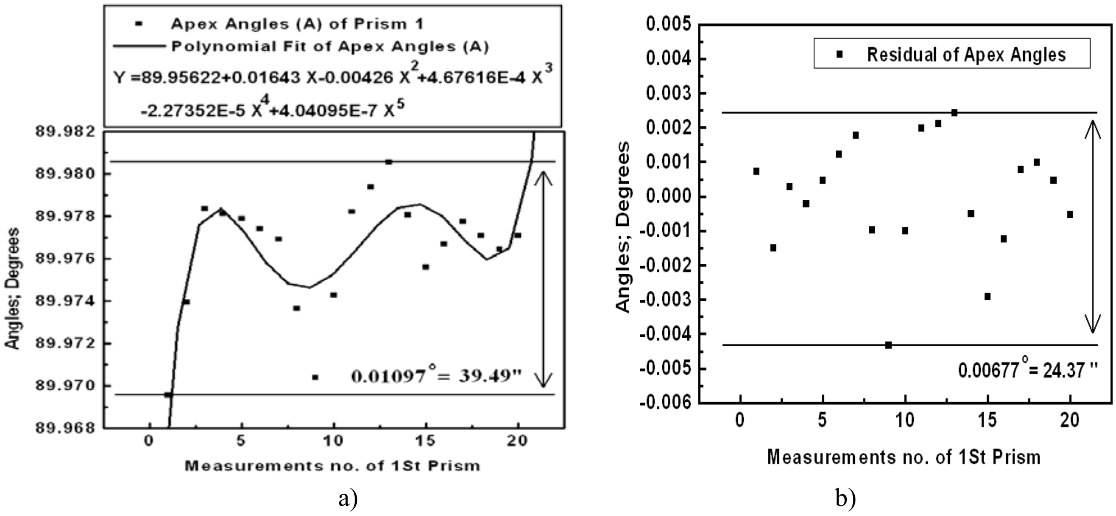


Fig. 11. Polynomial regression of results of apex angles of 1st prism; a) The 5th order equation fitted curve with an error value equal to 39.49 sec., b) Residuals of pol. Reg. of results with a reduced error value of 24.37 sec.

The correction by a model fitted to this curve and corrected for by the OCMI software is expected to give residual errors as shown in Fig.11.b. This correction would reduce the error of apex angle measurements from 39.49 sec. to 24.37 sec. which gives an improvement equal to 38.29 %.

5. UNCERTAINTY OF MEASUREMENTS

The uncertainty of measurements is evaluated in ISO standards applying two types of evaluation that are known as type A and type B. Type A uncertainty evaluation is performed applying statistical analysis of the series of observations. Type B uncertainty evaluation is realized by applying other methods than statistical methods. The combined uncertainty parameter includes all the systematic and random uncertainties [13].

Since the instrument works in the clean room's area in very good and stable environmental conditions, with temperature equal to (20 ± 0.5)°C and humidity equal to 60%. Therefore, systematic errors created due to effects of the environmental conditions in the area around the instrument are compensated by introducing the data on temperature, temperature gradient and air pressure to the instrument software, then the computer corrects the measured results. Systematic errors resulted from the mechanical and optical parts of the OCMI instrument are corrected during the measurements applying special software error correction based on the periodical calibration data of the instrument. Systematic errors resulting from the operator of the OCMI instrument are avoided by using the learning facility for programming the measurement procedure and running it in the automatic measurement mode. Therefore, the systematic uncertainties of the prism angles are corrected and only the random uncertainties are required to be evaluated. According to the method of the international Organization for Standardization (ISO) for evaluation of the uncertainty [13], the uncertainty of the angles measured by OCMI can be evaluated using the combined uncertainty that includes all individual uncertainties of the results given in Table 1. The individual uncertainty is calculated using the standard deviation of the mean of results as follows:

$$\mathrm{u_j} = \sqrt{\frac{1}{(n-1)}\sum_{j=1}^{n}(x_j - \bar{x})^2} \quad , \tag{6}$$

where: n - is number of measurements that equals 20, x_j - is the measured value, $\bar{x}$ - is the mean value.

The uncertainty of all the angle measurements can be evaluated due to ISO as the summation of the squares of all the individual uncertainties listed in Table 1., therefore:

$$u_c^2 = \left(u_{\mathrm{P1B1}}^2 + u_{\mathrm{P2B1}}^2 + u_{\mathrm{P3B1}}^2 + u_{\mathrm{P4B1}}^2 + u_{\mathrm{P1B2}}^2 + u_{\mathrm{P2B2}}^2 + u_{\mathrm{P3B2}}^2 + u_{\mathrm{P4B2}}^2 + u_{\mathrm{P1A}}^2 + u_{\mathrm{P2A}}^2 + u_{\mathrm{P3A}}^2 + u_{\mathrm{P4A}}^2\right), \tag{7}$$

where: P1~4 is prism number from 1 to 4, B1 is 1st prism base angle, B2 is 2nd prism base angle and A is apex angle of the prism.

$$u_c = \sqrt{\begin{matrix}[(6.5")^2 + (3.9")^2 + (3.7")^2 + (4.8")^2 + (4.7")^2 + (2.9")^2 \\ + (4.1")^2 + (3")^2 + (10")^2 + (4.4")^2 + (6.5")^2 + (6.5")^2]\end{matrix}} = \pm 0.00523^\circ = \pm 18.8"$$

Therefore, the combined uncertainty of the right angle prism measurements = ± 18.8”.

Since the results of prism angles follow the normal distribution, it follows that the uncertainty u_c evaluated statistically includes an interval of 68% of the measured results. The expanded uncertainty extends the confidence interval to 95% by multiplying the combined uncertainty by a coverage factor (k = 2, with confidence interval = 95%) [13], as follows.

The expanded uncertainty U = u_c x 2 = ±0.00523°. x 2 = ± 0.01046°. = ± 37. 66”.

The value of measurand = Mean of the results ± u_c x K.

For example, the values of the angles of the first prism are as follows:

B1 (1st prism base angle) = 44.99477° ± 0.01046° = 44° 59’41’’ ± 37.66’’ (at conf. level 95%).

B2 (2nd prism base angle) = 44.98582° ± 0.01046° = 44° 59’ 8.9’’ ± 37.66’’ (at conf. level 95%).

A (prism apex angle) = 89.97636° ± 0.01046° = 89° 59’ 3’’ ± 37.66’’ (at conf. level 95%).

The Optical Coordinate Measuring Instrument OCMI technique is a useful method for quick checking and testing due to its reliability and the ability to be operated automatically for the measurements of the prism angles and optical elements, specially during the process of the production that is of great importance.

6. CONCLUSIONS

1. The calibration of the angles of right angle prisms applying optical coordinate measuring instrument is discussed and the method is reliable, fast and accurate.
2. The expanded uncertainty of measurements = ± 37.66'' (at a confidence interval of 95%).
3. The technique is an optical non-contact method that is based on measuring the actual angles located between the three surface planes of the right angle prism.

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POMIARY KĄTOWE PRYZMATÓW PROSTOKĄTNYCH WYKORZYSTUJĄCE OPTYCZNĄ WSPÓŁRZĘDNOŚCIOWĄ MASZYNĘ POMIAROWĄ

Streszczenie

W tej pracy zaprezentowano nową metodę pomiarów kątów pryzmatów prostokątnych wykorzystującą optyczną współrzędnościową maszynę pomiarową. Jest to optyczna metoda bezkontaktowa, która opiera się na ogniskowaniu obrazów mierzonych elementów. Pomiary kątów elementów optycznych są bardzo ważne, ponieważ mają wpływ na dokładność technik pomiarowych. Uzyskane wyniki i ich niepewności potwierdzają poprawność metody.