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ANALYSIS OF A THREE-VOLTMETER MEASUREMENT METHOD DESIGNED FOR LOW-FREQUENCY IMPEDANCE COMPARISONS

The principle of the three voltmeter method designed for precise impedance comparison is presented. Generalization of the classical approach is proposed and analysed. Formulas for impedance parameters determination are derived and measurement uncertainty analysed for various kinds of comparisons, and next optimal measurement conditions specified. Important voltmeter parameters, such as random error and non-linearity error, are experimentally investigated. The presented results show that by using the method and commercially available apparatus it is possible to compare two impedances with a relative uncertainty of $(1\text{-}5)\times10^{-6}$ in the low frequency range.

Keywords: impedance measurements, impedance standards, voltage measurement, impedance comparison

1. INTRODUCTION

In recent years a great improvement of digital voltmeter parameters is observed, such as accuracy, linearity, stability and frequency range. This progress is particularly evident in DC measurements but also a great step in improving AC measurements has been made. Although the linearity of precision AC digital voltmeters is still lower than that of inductive voltage dividers (IVD's), it is high enough to be taken as a basis for development of impedance measurement systems more accurate than commercially available instruments. This is the case for the three-voltmeter method, already developed for particular purposes [1, 2, 3], but suitable to generalisation.

In the paper properties of the three-voltmeter method are analysed. It is shown that two major problems make it difficult to construct a precise system based on the method and limit the scope of its application. The first one follows from the demand of precise voltage measurement that one point must be at low potential. This cannot be accomplished directly for all three measured voltages. The second one is inevitable stray impedance of the lead connecting both compared standards: measurement impedance and a reference impedance standard. To solve the first problem Cabiati proposed a solution [1], which applies an inductive voltage divider (IVD) set to a value of 0.5. It is possible to measure all three voltages with one end at a low potential without loosing accuracy, and for the second one the special circuit design to inject a current which compensates the stray parameters of the leads.

In the paper a new generalised approach is proposed, which uses the IVD set to any value $k \in (0,1)$. This increases the method's flexibility and make it possible to find optimal measurement conditions for comparing various kinds of impedances, as for example inductance in reference to capacitance. The problem have been scrutinized and for various voltage ratios and various kinds of the compared impedances an optimal setting of the IVD has been found. Also a general measurement uncertainty analysis has been carried out and formulas derived for examination of particular measurement cases. Different hypotheses have been considered about the calibration of the voltmeters, such as: no special calibration of voltmeters and voltmeters with calibration of linearity. Important parameters of digital voltmeters: noise, linearity and stability have been investigated and values assessed.

2. THE PRINCIPLE OF THE METHOD

The basic, simplified scheme of the three-voltmeter method designed for impedance comparison is presented in Fig. 1a). The same sinusoidal current, $\boldsymbol{I} = I_m e^{j\omega t}$, is driven through two compared impedances: the impedance under test $\boldsymbol{Z}_x = R_x + jX_x$ (IUT) and the reference impedance standard $\boldsymbol{Z}_s = R_s + jX_s$.

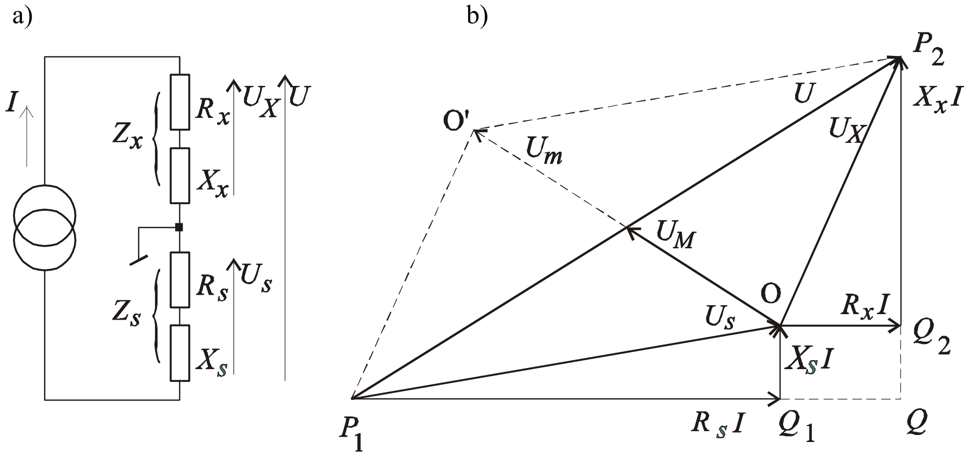

Fig.1. The scheme of three voltmeters method applied for impedance comparison:
a) the basic circuit; b) the vector diagram.

To determine the relation between the impedance $\boldsymbol{Z}_x$ (R_x, X_x) and the impedance $\boldsymbol{Z}_s$ (R_s,X_s), three voltages $U_s = |\mathbf{U_s}|$, $U_x = |\mathbf{U_x}|$, and $U = |\mathbf{U}|$ are measured. A vector diagram of these voltages is presented in Fig. 1b), for the case of linear impedances. Analysing the relation between the measured voltages U_s, U_x, U, the current $I = |\mathbf{I}|$ and the parameters R_x, X_x, R_s, X_s of the compared impedances we get the system of equations

$$\begin{cases} (R_s + R_x)^2 + (X_s + X_x)^2 = \alpha^2 (R_s{}^2 + X_s{}^2), \\ R_x{}^2 + X_x{}^2 = \alpha_x{}^2 (R_s{}^2 + X_s{}^2), \end{cases}$$

where $\alpha = U/U_s$ and $\alpha_x = U_x/U_s$ are relative voltages, referred to the voltage U_s. By substituting the second equation to the first one we obtain the linear equation, with respect to R_x and X_x:

$$R_s R_x + X_s X_x = \alpha_B (R_s^2 + X_s^2), \tag{1}$$

where the coefficient α_B is defined by the formula

$$\alpha_B = (\alpha^2 - \alpha_x^2 - 1)/2. \tag{2}$$

To obtain a second linear equation we compare the area A of the triangle P_1OP_2 calculated, by the Heron's formula, directly from its legs, to its area calculated by subtracting from the area of the right triangle P_1QP_2 the areas of the right triangles P_1Q_1O, OQ_2P_2, P_1OP_2, and the rectangular OQ_1QQ_2. From this, after transformations, we get the second linear equation

$$-X_sR_x + R_sX_x = \alpha_A(R_s^2 + X_s^2), \quad (3)$$

where $\alpha_A = 2A/U_s^2$ is a coefficient proportional to the area A, the value of which can be calculated from the formula

$$\alpha_A = \frac{1}{2}\sqrt{\left[1-(\alpha-\alpha_x)^2\right]\left[(\alpha+\alpha_x)^2-1\right]}. \quad (4)$$

The Equations (1) and (3) form a system of two linear equations and its solution provides formulas

$$\begin{cases} R_x = \alpha_B R_s - \alpha_A X_s \\ X_x = \alpha_A R_s + \alpha_B X_s \end{cases} \quad (5)$$

for the parameters R_x and X_x, determination provided the parameters R_s and X_s of the reference impedance and voltages U_s, U_x, U, are known. The formulas (5) are similar to those obtained by Cabiati et al. [1] and also resemble to a certain degree the balance equations of a Wheatstone type bridge. In the Wheatstone bridge, coefficients α_A and α_B are equal to the ratio of voltages, which are equal to ratios of impedances, whereas in the three-voltmeter method the coefficients α_A and α_B are functions of squared ratio of voltages. As the result of this quadratic relation the comparison error depends on many agents and to obtain the highest accuracy the optimal measurement conditions should be assessed. This will be done farther in the paper.

3. THE ACTUAL CONFIGURATION OF MEASUREMENT SYSTEMS

To apply this method for precise impedance comparison, two major problems should be solved: the first one is precise measurement of the voltage U affected by a large common mode voltage and the second one is the influence of the stray impedance of the lead connecting the compared impedances. To solve the first problem Cabiati [1] connected the IVD between points P_1 and P_2, Fig.1, with voltage output to input ratio, k, set to 0.5 and measured the voltage U_M between the output tap of the IVD and the ground (instead of the voltage U).

Let $\alpha_M = U_M/U_s$ and $\alpha_m = U_m/U_s = 2\,U_M/U_s = 2\alpha_M$ be the relative voltages referred to the voltage U_s. To determine the relation between α and α_M (or between α and α_m which is more convenient for further analysis) we compare the area A of the P_1OP_2 triangle to the area of the triangle P_1OO'. Both areas are calculated by using Heron's formula. After some transformations we obtain the equation

$$\alpha^2 = -\,\alpha_m^2 + 2\,(1 + \alpha_x^2).$$

Next, by substituting this formula to Eq. (2) we get a new formula for the α_B coefficient

$$\alpha_B = (1 + \alpha_x^2 - \alpha_m^2)/2. \quad (7)$$

Similarly by using the Eq. (7) in (4) we get the new formula for α_A

$$\alpha_{\mathrm{A}} = \frac{1}{2}\sqrt{\left[1-(\alpha_m-\alpha_x)^2\right]\left[(\alpha_m+\alpha_x)^2-1\right]}, \tag{8}$$

which is similar to (4). The only difference is that the coefficient α has been substituted by α_m.

To achieve high accuracy measurements, the compared impedances should work under conditions specified in the definition of the four port impedances, introduced by Cutkosky [4]. It is considered to be the most rigorous definition for shielded impedances or admittances equipped with four coaxial connectors [5]. In the four-port impedance, the voltage is defined at the high-voltage port in open circuit condition while the current is defined at the low-voltage current port whose voltage is equal to zero. One way of solving this problem, suggested by Cabiati [1], is based on using a current injection near the middle section of the yoke, which connects the compared impedances, to compensate the voltage drops on the conductors. This solved the problem but was inconvenient for operation and needed complicated manual adjustment. Poliano and others [2, 3] developed the solution which operates automatically, preserving high accuracy. However the solution is complicated.

4. GENERALIZATION OF THE CIRCUIT WITH IVD

In the solution presented in [1] the IVD has the ratio of voltages k set to the value equal to $k = 0.5$ (the voltage U is divided by 2). We shall generalize this condition by letting the IVD to be set to any value of k between 0 and 1 and measuring the voltage U_k instead of U_M (see Fig. 2). Now the task is how, knowing the voltages U_s, U_x, U_k and the setting of k.

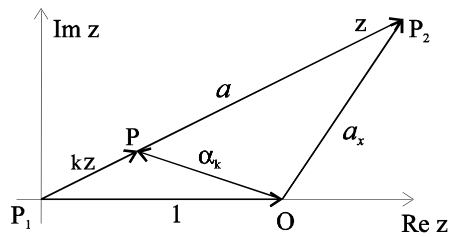

Fig.2. Geometrical presentation of the generalised three voltmeter method.

the IVD (or what is equivalent knowing relative voltages $\alpha_x = U_x/U_s$, $\alpha_k = U_k/U_s$, and k), to determine values of the coefficients α_A and α_B. The geometrical presentation of the problem, in the Gaussian plane, is shown in Fig. 2. The task can be stated geometrically as follows: given $|P_1O| = 1$, $|OP_2| = \alpha_x$, $|OP| = \alpha_k$ and $k = |P_1O|/|P_1O|$ find the relative voltage $|P_1P_2| = \alpha$.

The complex number $\boldsymbol{z}$ satisfies equations

$$|\boldsymbol{z} - 1| = \alpha_x , \tag{9}$$
$$|k\boldsymbol{z} - 1| = \alpha_k . \tag{10}$$

Solving this system of equations of complex variables, with respect to $\alpha = |\boldsymbol{z}|$ we obtain

$$\alpha^2 = \frac{1}{1-k}\alpha_x{}^2 - \frac{1}{k(1-k)}\alpha_k{}^2 + \frac{1}{k}. \tag{11}$$

Next, by setting this value to the Eq. (2) we get the general formula for α_B

$$\alpha_B = \frac{1}{2}\left[\frac{k}{1-k}\alpha_x{}^2 - \frac{1}{k(1-k)}\alpha_k{}^2 + \frac{1-k}{k}\right]. \tag{12}$$

Similarly, applying the Eq. (12) to (8), after some transformation, we get the new general formula for $\alpha_A{}^2$

$$\alpha_A{}^2 = \alpha_x{}^2 - \frac{1}{4}\left[\frac{k}{1-k}\alpha_x{}^2 - \frac{1}{k(1-k)}\alpha_k{}^2 + \frac{1-k}{k}\right]^2. \tag{13}$$

It can be easily checked that by setting for k the value $k = 0.5$ we get the formulas (7) and (8) as a particular case of the generalized approach.

The general formulas (5) for determination of the parameters R_x and X_x remain the same as for $k = 0.5$. The only difference is that now the coefficients α_A and α_B are calculated from generalized formulas (12) and (13).

5. MEASUREMENT OF IMPEDANCES

The circuit is suitable for impedance comparisons. We shall now determine formulas for measurement of particular impedances. We assume that the reference standard is the resistance and derived formulas for determination of resistors, inductors and capacitors. The measured impedance, IUT, will be presented in the two element series equivalent circuits:

resistor: $\boldsymbol{Z_x} = R_x + j\omega L_x = R_x(1 + j\omega\tau_x)$, where $\tau_x = L_x/R_x$ is the time constant,
inductor: $\boldsymbol{Z_x} = j\omega L_x + R_x = j\omega L_x(1 - j/Q_x)$, where $Q_x = \omega L_x/R_x$ is the storage factor,
capacitor: $\boldsymbol{Z_x} = 1/j\omega C_x + R_x = (1 + j D_x)/j\omega C_x$, where $D_x = \omega R_x C_x$ is the dissipation factor.

The impedance of the standard resistor is equal to $\boldsymbol{Z_s} = R_s + j\omega L_s = R_s(1 + j\omega\tau_x)$, where $\tau_s = L_s/R_s$ is the time constant of the reference resistor. It can be shown that parameters of the IUT can be determined from the formulas:

resistor:

$$R_x = (\alpha_B - \alpha_A\omega\tau_s)R_s, \tag{14}$$

$$\tau_x = \frac{1}{\omega}\frac{\alpha_A + \alpha_B\omega\tau_s}{\alpha_B - \alpha_A\omega\tau_s}, \tag{15}$$

inductor:

$$L_x = (\frac{1}{\omega}\alpha_A - \alpha_B\tau_s)R_s, \tag{16}$$

$$Q_x = \frac{\alpha_A + \alpha_B \omega \tau_s}{\alpha_B - \alpha_A \omega \tau_s}, \qquad (17)$$

capacitor:

$$C_x = \frac{1}{\omega(\alpha_B - \alpha_A \omega \tau_s) R_s}, \qquad (18)$$

$$D_x = \frac{\alpha_B - \alpha_A \omega \tau_s}{\alpha_A + \alpha_B \omega \tau_s}. \qquad (19)$$

The next problem is to assess the uncertainty of determination of these parameters as functions of uncertainties of the voltage measurements .

6. UNCERTAINTY ANALYSIS

The parameters R_x and X_x of the IUT are determined from the formulas (5) and its uncertainty depends mainly on uncertainties of determination of the coefficients α_A and α_B. The coefficients α_A and α_B are determined from:

a) measurements of voltages U_s, U_x, and U (or U_m, U_M),

b) measurements of voltages ratios α_x and α (or α_m, α_M).

The second case occurs when the linearity of voltmeters is calibrated, e.g. in reference to the IVD. We shall derive formulas for uncertainties for both mentioned conditions and next apply them for particular measurements.

The following notation is used: the absolute uncertainty of a quantity x is denoted by Δx and the relative, by $\delta x = \Delta x/x$. We derive formulas for uncertainties for the general configuration first and analyze the particular case when $k = 0.5$.

6.1. Uncertainty of determination of α_A

First, we consider case a) when voltages are measured. To assess the upper limit of the relative uncertainty $\delta\alpha_A$, we substitute $\alpha_x=U_x/U_s$ and $\alpha_x=U_m/U_s$ into Eq. (13), calculate partial derivatives with respect to U_x, U_s and U_m and divide both sides of the equation by α_A. From this we get the formula for the upper limit of relative uncertainty $\delta\alpha_A$.

To analyze how this uncertainty depends on the values of the compared impedances parameters it is essential to express formulas in terms of the compared impedance parameters, R_x, X_x and R_s (we assume that the reactance X_s of the reference impedance $\boldsymbol{Z_s}$ is negligible in comparison to R_s). It can be proved that, under these conditions, the following equations hold:

$$\begin{aligned} &\alpha_A = x, \\ &\alpha_x^2 = r^2 + x^2, \\ &\alpha_m^2 = (1 - r)^2 + x^2, \\ &\alpha_k^2 = [k(1 + r) - 1]^2 + k^2 x^2 \\ &\alpha_B = r, \end{aligned} \qquad (20)$$

where $x = X_x/R_s$ is relative reactance referred to the resistance R_s and $r = R_x/R_s$ is relative resistance referred to R_s.

By substituting them to the equation for the relative uncertainty $\delta\alpha_A$ we get the following formula

$$\delta\alpha_A \leq \frac{1-k(1+r)}{1-k}(1+\frac{r^2}{x^2})\delta U_x + r(1+\frac{(1-r)^2}{x^2})\delta U_k + (1+r\frac{1-k-kr}{kx^2})\delta U_s, \tag{21}$$

for its upper limit. In the case of independent measurements we can assess the value of $\delta\alpha_A$ from the equation

$$\delta^2\alpha_A = \left(\frac{1-k(1+r)}{1-k}(1+\frac{r^2}{x^2})\right)^2 \delta^2 U_x + r^2\left(1+\frac{(1-r)^2}{x^2}\right)^2 \delta^2 U_k + \left(1+r\frac{1-k-kr}{kx^2}\right)^2 \delta^2 U_s. \tag{22}$$

The respective formulas for case b), when α_x, and α_m are measured, can be obtained from case a) simply by substituting $U_s = 1$ to the above formula. Then we get: $\alpha_x = U_x/U_s = U_x$ and $\alpha_m = U_m$, and in consequence $\delta U_s = 0$, $\delta U_x = \delta\alpha_x$ and $\delta U_m = \delta\alpha_m$, where $\delta\alpha_x$ and $\delta\alpha_m$ are uncertainties of determination of the voltage ratios α_x and α_m, respectively.

6.2. Uncertainty of determination of α_B

Using the same technique we can derive formulas for the uncertainty $\delta\alpha_B$ for the case a) when voltages are measured and from this we get the formula

$$\delta^2\alpha_B = \left(\frac{k}{1-k}\frac{\alpha_x^2}{\alpha_B}\right)^2 \delta U_x + \left(\frac{1}{k(1-k)}\frac{\alpha_k^2}{\alpha_B}\right)^2 \delta^2 U_k + \left(\frac{k}{1-k}\frac{\alpha_x^2}{\alpha_B} - \frac{1}{k(1-k)}\frac{\alpha_k^2}{\alpha_B}\right)^2 \delta^2 U_s. \tag{23}$$

Now having these results we shall analyse particular cases of comparisons: resistance versus resistance (R_x v. R_s). Inductance versus resistance (L_x v. R_s) and capacitance versus resistance (C_x v. R_s). We now discuss the problem in turns.

6.3. Uncertainty of comparison R_x v. R_s

The value of R_x is calculated from formula (5). For the error analysis we consider only the main part of the right hand side of this equation, which means that $R_x \cong \alpha_B R_s$. Thus the relative uncertainty of determination of δR_x of R_x depends mainly on uncertainty $\delta\alpha_B$ which is given by Eq. (23). For typical conditions, when the resistor under test has a small time constant τ_x, and we can assume that $x = X_x/R_s << 1$, we get the assessment

$$\delta R_x = \sqrt{\delta^2 R_s + \left(\frac{1}{r} - 2 + r\right)^2 \delta^2 U_m + r^2\delta^2 U_x + \left(\frac{1}{r} - 2\right)^2 \delta^2 U_s}. \tag{24}$$

A graph of the relative uncertainty of comparison δR_x of the R_x to R_s is for the case when voltages U_s, U_x and U_M are measured ($k = 0.5$) is presented in Fig. 3.

As seen, the uncertainty has a minimum for r equal approximately to 0.5. If, for instance, we assume that the comparison error should be smaller than 1.4 δU then the ratio $r = R_x/R_s$ should be within the interval (0.35, 1.5).

6.4. Uncertainty of comparison of Lx v. Rs

The value of inductance is calculated from Eq. (14). We assume that the reference standard R_s has a very small (negligible) time constant, then the second term in the parentheses of the right hand side of (14) is very small and we can neglect it for the error analysis . Thus the value of inductance is determined from the simplified formula

$$L_x = \alpha_A R_s/\omega \qquad (25)$$

and the relative uncertainty δL_x is approximately equal to $\delta\alpha_A$ (provided that $\delta\omega$, δR_s << $\delta\alpha_A$).

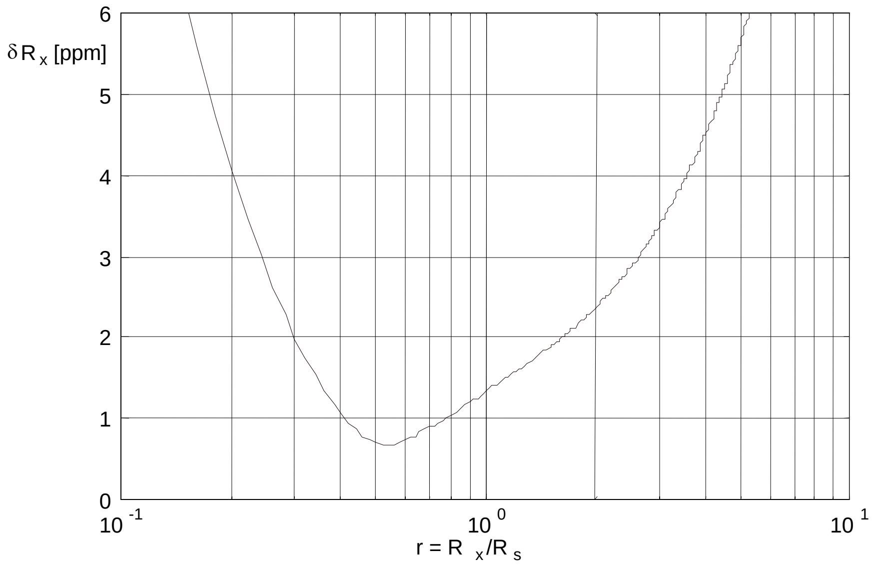

Fig. 3. Relative uncertainty δR_x of R_x v. R_s comparison, as a function of the of compared resistances ratio $r = R_x/R_s$.

We shall analyze the dependence of $\delta\alpha_A$ on the ratio r of the compared elements. If the storage factor Q_x is substantially larger than one ($Q_x >> 1$) and the reactance of the IUT, X_x is of the same order of magnitude as R_s, then the ratio $r = R_x/R_s$ is much smaller than 1 and the formula (22) for the uncertainty simplifies to the form

$$\delta\alpha_A = \sqrt{r^2\delta^2 U_m + \delta^2 U_x + \delta^2 U_s}\,. \qquad (26)$$

The uncertainty $\delta\alpha_A$ depends mainly on uncertainties of measurements of voltages U_x and U_s (or α_x) . If the storage factor Q_x is substantially smaller than one ($Q_x << 1$ and $r << 1$) then the first term

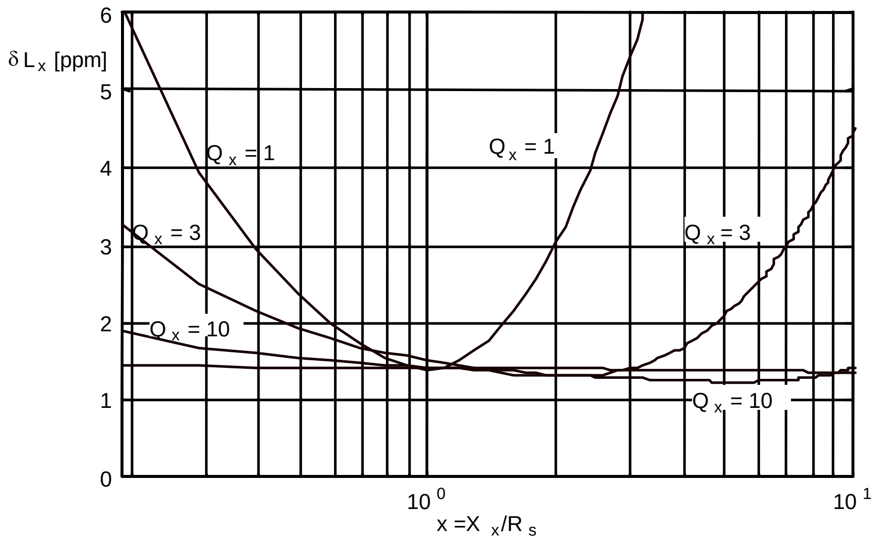

Fig. 4. Uncertainty δL_x of inductance determination as a function of $x = \omega L_x/R_s$ for various values of the storage factor Q_x and for $\delta U_s = \delta U_x = \delta U_M = 1$ppm.

in the square root of the above formula vanishes. The uncertainty of determination of inductance L_x is approximately equal to the uncertainty of determination of the ratio $\alpha_x = U_x/U_s$ and does not depend on the value of R_s. However for inductance standards measured at low frequencies the storage factor Q_x is small. For instance, for the often used GenRad inductance standards, type 1482, the value of the storage factor Q_x is about 10 for a frequency of 1 kHz and can be even smaller than 1 for 100Hz. The uncertainty δL_x depends, in this case, on the value of the reference standard R_s, Fig 4, and we can observe that for the large Q_x values, the error is approximately constant - it does not depend on $x = X_x/R_s$ but depends significantly on x for smaller Q_x values. The minimal measurement error is obtained for $x \cong 1$. Thus for the low values of storage factors it is important to refer the inductance L_x to such resistor R_s whose value satisfies the condition $R_s \cong \omega L_x$.

6.5. Uncertainty of comparison of *Cx* v. *Rs*

The formula for the determination of capacitance C_x is given by (18). For the error analysis we shall consider only the main part of the denominator, then $C_x \cong 1/(\omega\alpha_B R_s)$. The relative uncertainty δC_x is approximately equal to $\delta\alpha_B$ (uncertainty $\delta\omega$ is negligible and R_s is the reference standard value). Dissipation factor, D, of real capacitors satisfies the condition $D \le (2\text{-}4)\times 10^{-4}$ and from this we have $r << 1$ and $r << x$. Then the value of uncertainty, δC_x, for the low value of the dissipation factor, is simply given by the formula $\delta C_x \cong \delta\alpha_B \cong \delta\alpha_x$ (see Eq. (12)).

From the above analysis we can draw two major conclusions: that in the case of capacitance measurements in reference to resistance the voltage U_m (and U_k) can be measured with much lower accuracy than the other voltages, and that the uncertainty of determination of $\delta\alpha_B$ depends directly on uncertainties of determination of voltages U_x and U_s (or the ratio α_B).

7. OPTIMIZATION OF MEASUREMENT CONDITIONS

The uncertainty of comparison depends on the value of the IVD setting *k* (see (22) and (23)). We shall analyze now how to chose a value of *k* in particular measurements.

7.1. Choice of optimal *k* in inductance and capacitance measurements

The uncertainty of inductance and capacitance measurements is approximately equal to uncertainty $\delta\alpha_A$ of determination of the coefficient α_A. In measurements of capacitors with a low dissipation factor, D_x, values and inductors with a high value of the storage factor Q_x, the coefficient *r* is very small, $r << 1$, and uncertainty $\delta\alpha_A$ simplifies to the formula

$$\delta\alpha_A \approx \sqrt{\delta^2 U_x + \delta^2 U_s}\ , \qquad (27)$$

which does not depend on *k*. However, for inductors measured at low frequencies, the first part in formula (26) is quite significant and its important to determine the optimal value for *k* for each particular case.

In Figure 5 the uncertainty δL_x of L_x is presented for a 10 mH GenRad inductance standard, referred to 20Ω, 60Ω and 100Ω resistance standards, R_s, for measurements at a frequency of f = 1kHz. The uncertainty changes substantially with the value of *k* and for the reference standard R_s = 60Ω, the minimal measurement error is obtained for $k \cong 0.87$. For a lower frequency (f < 1kHz) the influence of the value of *k* on the measurement error is getting larger and it is essential to calculate an optimal value of *k* for each R_s.

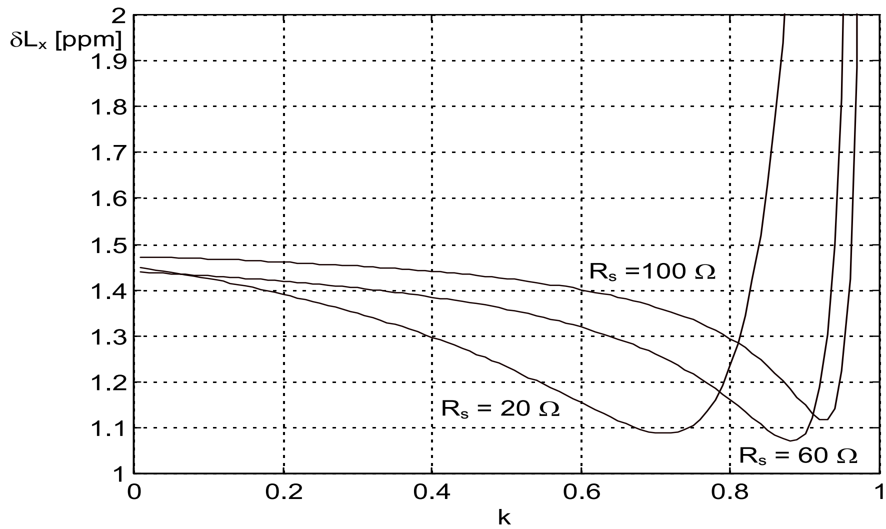

Fig. 5. Uncertainty δL_x as a function of the IVD setting *k* for different values of reference R_s standard.

7.2. Choice of optimal *k* in resistance measurements

The uncertainty of resistance measurements is approximately equal to the uncertainty of determination of the α_B coefficient . For comparison of R_x v. R_s, with small values of time constants of the compared standards we can neglect the value of the ratio $x = X_x/R_s$. The plot of measurement uncertainty as a function of k for various values of $r = R_x/R_s$ is presented in Fig. 6. The results

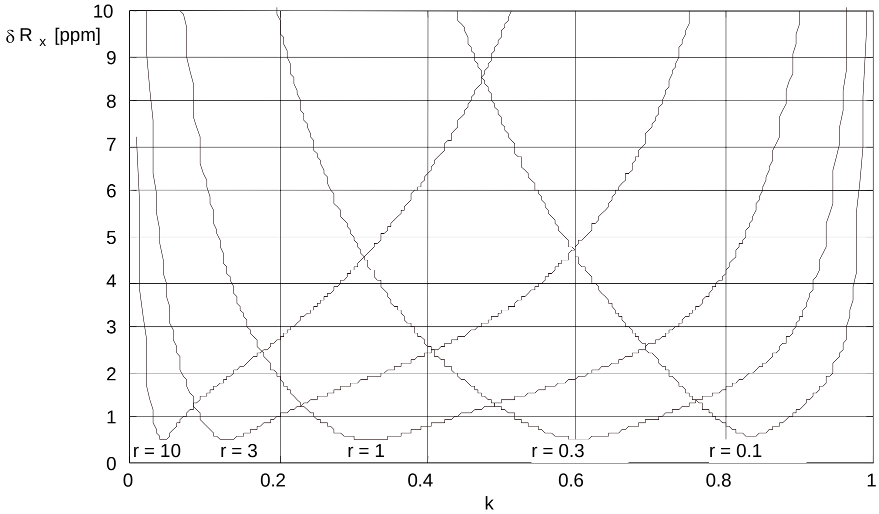

Fig. 6. Uncertainty of R_x v. R_s comparison as a function of the IVD setting k for various ratios $r = R_x/R_s$.

show a very strong dependence of uncertainty on the value of k and this means that the optimal value of k should be calculated for each measurement and the IVD setting. A similar analysis should be performed when large capacitances are measured.

8. UNCERTAINTY OF VOLTAGE MEASUREMENT

The accuracy of measurements is determined by uncertainty of voltage measurements. This depends on the parameters of voltmeters and the generator. The most important factors are: stability of voltmeters and generators and linearity of voltmeters. In the case of generators we need high stability, particularly for the configuration in which. only one voltmeter is used to measure all three voltages. Two voltmeters of type “AC Standard 5790A” and one generator “Calibrator 5700A”, manufactured by FLUKE were examined. Most measurements were made in the 2.2 V range and at a frequency of 1000 Hz.

8.1. Stability

Two kinds of stability have been examined: mutual stability of one voltmeter v. another one and mutual stability of a voltmeter v. the calibrator. From the obtained results the following conclusions can be drawn:

a) relative difference between successive readings does not exceed ±5 ppm,
b) relative difference between "moving average" of 10 readings usually does not exceed ± 1.2 ppm,
c) relative difference between moving average of 100 readings usually does not exceed ± 0.3 ppm
d) drift of moving average of 100 readings usually does not exceed ± 0.3 ppm/minute.

Sometimes it happens that there is a bigger change in successive readings. In this case we should remove these results as outliers.

Mutual stability of the calibrator and voltmeter was examined for the 2.2 V range. From these measurements the following conclusions can be drawn: the system (particularly the calibrator) should operate at least 24 h before the measurements. It is not enough to keep it in the “stand by” mode. After switching from the “stand by” mode to “operate” mode the output of the calibrator drifted for many hours and during this time quick changes of readings resulting in ± 2 ppm "jumps" of 100 readings average, have been observed. It was probably due to the internal system which controls the level of the calibrator output. Such jumps were rarely observed after 24 hours work and the average of 500 readings was very stable - under 0.2 ppm/4 hours.

To get the highest accuracy of measurements the following procedure is recommended:

a) set the system in operating mode for at least 24 h and observe if successive readings are stabilised,
b) start the measurement from the highest voltage of Ux,U_0 and Us, say Ux and take 500 readings of each voltage measurement,
c) during measurements, control the jumps of readings and in case they should occur remove them from the data,
d) return to the measurement of the first voltage, Ux, and if the average of 500 readings did not change more from the first series than an assumed value, say 1 ppm, the measurement results can be accepted.

8.2. Linearity

The linearity error of the voltmeter was checked in reference to the IVD, Model 73 manufactured by ESI, with a relative uncertainty of 0.5 ppm. Two voltmeters were examined, AC STANDARD manufactured by FLUKE, at 1 kHz frequency and Umax = 2.2V range and (some measurements were made also at lower ranges) the following results have been obtained:

a) The non-linearity error, in the voltage range (0.6 Umax to Umax) referred to the measured value, is smaller than 2 ppm. For the lower voltage value the error is greater and for U/Umax = 0.3 can be as big as 6.5 ppm.
b) Non-linearity errors of both checked voltmeters have a similar shape and the difference between them, for the range 2.2 V, is smaller than 0.2 ppm.
c) For the range of 220 mV the non-linearity error is greater and can be as big as 30 ppm.
d) The non-linearity error during 5 days of examination did not change substantially. In the voltage range (0,6 -1)Umax the observed change was smaller than 0.3 ppm.

It is worth to notice that the non-linearity error of the examined voltmeters, for the range of 2.2 V, is smaller than 2 ppm. Thus the error of voltmeters is about 4-5 times greater than the non-linearity error of typical IVD’s.

9. CONCLUSIONS

The presented analysis of the three-voltmeter method and investigation of commercially available instruments proved that the method is suitable for impedance comparison with relative uncertainty up to a few ppm. To obtain such high accuracy the circuit should be optimized by: using a proper value of the reference standard, $\boldsymbol{Z_s}$, setting an optimal value, k, of the IVD; warming up the apparatus during at least 24 hours before measurements and using an average of least 500 readings with outliers removed.

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ANALIZA METODY TRZECH WOLTOMIERZY PRZEZNACZONEJ DO KOMPARACJI IMPEDANCJI W ZAKRESIE MAŁYCH CZĘSTOTLIWOŚCI

Streszczenie

Przedstawiono podstawy metody trzech woltomierzy przeznaczonej do precyzyjnych komparacji impedancji. Opracowano uogólnienie klasycznego układu i przeanalizowano jego własności. Wyprowadzono wyrażenia pozwalające na wyznaczanie parametrów impedancji i analizowano niepewność pomiarów dla różnych rodzajów komparacji. Określono stąd optymalne warunki pomiarów. Ważne parametry woltomierzy, takie jak błąd losowy i błąd nieliniowości były eksperymentalnie badane. Przedstawione wyniki wskazują, ze przy użyciu metody trzech woltomierzy z wykorzystaniem dostępnej w handlu aparatury pomiarowej, jest możliwe dokonywanie komparacji impedancji, w zakresie niskich częstotliwości, ze względną niepewnością na poziomie $(1\text{-}5)\times10^{-6}$.