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# SYSTEM OREIENTED MATHEMATICAL MODEL OF SINGLE MEASUREMENT RESULT

Measurements in a system are performed automatically by using data acquisition cards typically consisting of an amplifier, a sample/hold circuit and an analog-to-digital converter. The results obtained from these cards are processed by a program. The processing algorithms are often of sophisticated numerical structure and, in this situation, the determination of inaccuracy of the system output data needs building a system error model. The base of the error model construction should be a model of a single measurement result delivered at the output of the card. The paper presents a model which has been obtained on the basis of an analysis of the quantization process consisting in a direct comparison of the measured quantity with a standard of quantum character. In a measuring system the quantization is realized by an AD converter, which measures a sample of a time-varying input quantity. The assumption that the sampling is performed at any moment enables obtaining the model described in probabilistic categories, which may be the basis of the uncertainty calculation of the system output data.

Keywords: measurement system, data acquisition card, single measurement result model, error model, quantization

## 1.INTRODUCTION

The operation of modern measuring systems can be divided into two main parts commonly called data acquisition and data processing. Data acquisition consists in measuring input quantities of a system and collecting the digital measurement results in the memory of microprocessors or microcomputers. The collected data are then processed by different kind of algorithms in order to obtain final results in a form suitable for visualization and transmission to actuators when the system controls an object. In both cases it is necessary to know how accurate, or calling it better – inaccurate the final results are. This necessity should be formally expressed as a requirement that every number being the final result has to be provided for a second number which describes the inaccuracy of the first one. Taking into account that all operations of a system are performed automatically, the inaccuracy descriptor should be calculated by the system itself, so one may demand from the system that it is capable of self-determining its inaccuracy.

Such an ability of a system may be achieved when all measurement results obtained in the system are described in modeling categories, i.e. every result is a composition of a number being the measurement result and other component modeling factors which influence on the inaccuracy of this result. Traditionally, these factors are expressed as errors generally described as random when one takes into account that the system work is initiated by external events of random character.

The basis of determination of such a model is the metrological analysis of instruments used in systems to obtain measurement results. The most popular one is the so called data acquisition card applied in computers and its equivalent in modular systems, i.e. the ADC module. Both kinds of instruments are of the same construction, shown in Fig. 1, and they are called measuring cards in this paper.

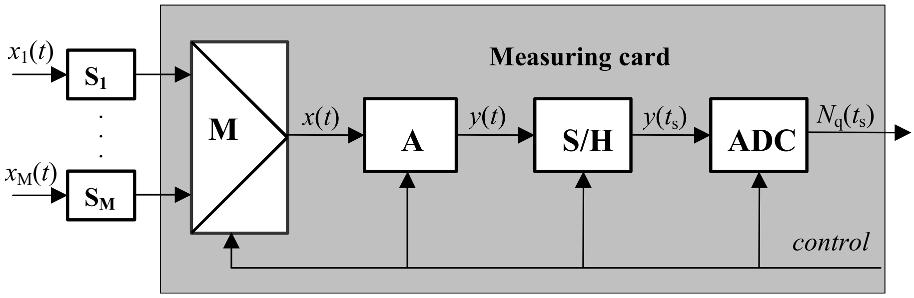


Fig. 1. Basic scheme of measurements in a system.

Figure 1 shows the scheme of the measuring card when $M$ input quantities $x_1(t)$, …, $x_M(t)$ are measured by the same card. They are described as varying in time because practically there are no quantities constant in time – the classification of a quantity depends on the time of its observation (measurement). Every quantity is individually converted by its sensor, denoted by **S**, to an electrical quantity – more often it is a voltage signal. The converted signals are commuted by an analog multiplexer **M** what means that in its chosen state only one selected input quantity may be measured. To simplify considerations, one assumes that the multiplexer state does not change, so the measured input quantity is only one and it is denoted as $x(t)$. It is converted to a voltage signal which is conditioned by the programmable amplifier **A**, i.e. an amplifier with digitally changed amplification coefficient, to a voltage level suitable for the range of the analog-to-digital converter **ADC**. The conditioned signal $y(t)$ is sampled by the **S/H** circuit at the selected moment $t_s$, $s$ is the symbol of the sampling moment, and held for a time necessary to convert the sample by the **ADC** to a number $N_q(t_s)$.

It is important to emphasize that AD conversion is a **measurement process** performed on the quantization principle. An analysis of this process is the basis of providing means of description of the whole measurement chain shown in Fig.1 and enables building a model of a measurement result obtained at its output. The starting point of building such a kind of model is the selection and description of the error sources arising in the chain when a time-varying quantity is measured [1].

## 2.MEASUREMENT MODEL OF A QUANTIZATION RESULT

Quantization, from the measurement point of view, is the process of direct comparison of the measured quantity with the sum of the same kind elementary measurement standards called quantum. In voltage AD converters, the quantum sum is usually obtained by using a voltage divider built from resistors of the same value $R$, or the values $R$ and $2R$ [7]. The quantizer scheme, in the simplest form, is shown in Fig. 2.

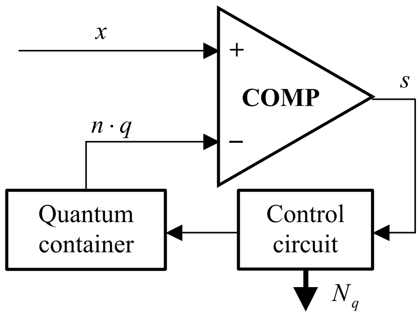


Fig. 2. Basic scheme of a quantizer.

The quantized value of $x$ is compared with the sum of $n$ quantum $q$ by the comparator **COMP**. Let us assume that the state of comparator $s = 1$ if $x - nq > 0$, in the other case $s = 0$. The task of the control circuit is to collect a minimum number of quantum for $s = 0$. Let us denote this number as $N_q + 1$, where $N_q$ is indication of the quantizer being the number of quantum assigned to the value of the measured quantity $x$, as shown in Fig. 3.

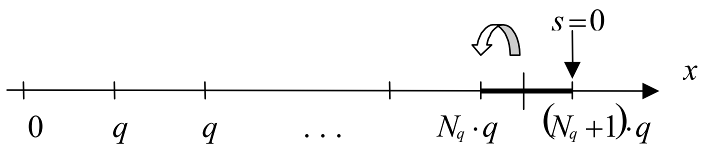


Fig. 3. Interpretation of a quantization result.

Accordingly with the interpretation, given by Fig. 3, the quantization process consists in the assignment of an interval to the measured quantity, which is caused by the quantum character of the standard used in this process. Therefore, one can write that the true value of the measured quantity fulfills the inequality:

$$N_q q < x \leq \left(N_q + 1\right) q \,. \tag{1}$$

The Equation (1) describes the relation between values of the measured quantity $x$ and values of the number $N_q$ obtained as the result of quantization for $x$ changes in a working range of $[0, x_{max}]$, for which $N_q$ takes integer values from 0 to $N_{max}$. Therefore, Eq. (1) may be written as a function:

$$N_q = ent\left(\frac{x}{q}\right), \tag{2}$$

where *ent* is the symbol of the function *entier* which delivers the integer part of its argument. The graphical form of Eq. (2), i.e. the quantizer characteristic, is shown in Fig. 4a.

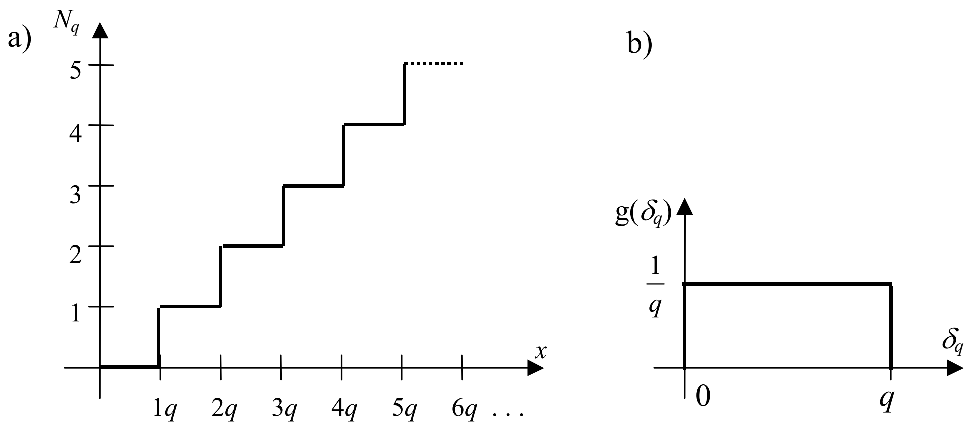


Fig. 4. a) Characteristic of the quantizer, b) distribution of the quantization error.

To simplify further considerations, let us denote:

$$\breve{x} \equiv N_q \, q \,, \tag{3}$$

where $\breve{x}$ is the quantizer indication expressed in units of the measured quantity. Introducing the relation (3) to the Eq. (1) one obtains:

$$\breve{x} < x \leq \breve{x} + q \tag{4}$$

and then

$$0 < x - \breve{x} \leq q\,. \tag{5}$$

Let us define the expression $x - \breve{x}$ as a quantization error:

$$\delta_q := x - \breve{x}\,, \tag{6}$$

since it describes how much the measured value differs from the suitable indication of the quantizer. After introducing the definition (5) to the Eq. (4), we have:

$$0 < \delta_q \leq q\,, \tag{7}$$

what means that, in the considered situation, values of the quantization error change in limits from 0 to $q$. If one takes into account that all values of the measured quantity may be treated as equally probable in its working range, the quantization error is described in probabilistic categories. For the quantizer characteristic as in Fig. 4a, its probability density function $g(\delta_q)$ (the error distribution ) is shown in Fig. 4b.

The expected value of the error distribution as in Fig. 4b

$$E(\delta_q) = \int_{-\infty}^{+\infty} \delta_q g(\delta_q) d\delta = 0.5q \tag{8}$$

is different from 0 what means that the measurement result is burdened by a systematic error. This error can be corrected by replacing the indication $\breve{x}$ by the value:

$$\hat{x} = \breve{x} + 0.5q\,, \tag{9}$$

called an evaluation of the measured result (i.e. measured result after correction of systematic error). After introducing Eq. (9) to inequality (5) one obtains $0 < x - (\hat{x} - 0.5q) \leq q$ and then

$$-0.5q < x - \hat{x} \leq 0.5q\,. \tag{10}$$

One achieves the same effect as described by Eq. (10) by adding 0.5 to $N_q$. The characteristic of such a kind of quantizer is described by the equation:

$$N_q = ent\left(\frac{x}{q} + 0.5\right) \tag{11}$$

and shown in Fig. 5a. The suitable quantization error distribution is presented in Fig. 5b. For the next considerations one assumes that all analyzed errors, i.e. both the quantization error and other ones, are described by probability density functions with the expected values equal to 0.

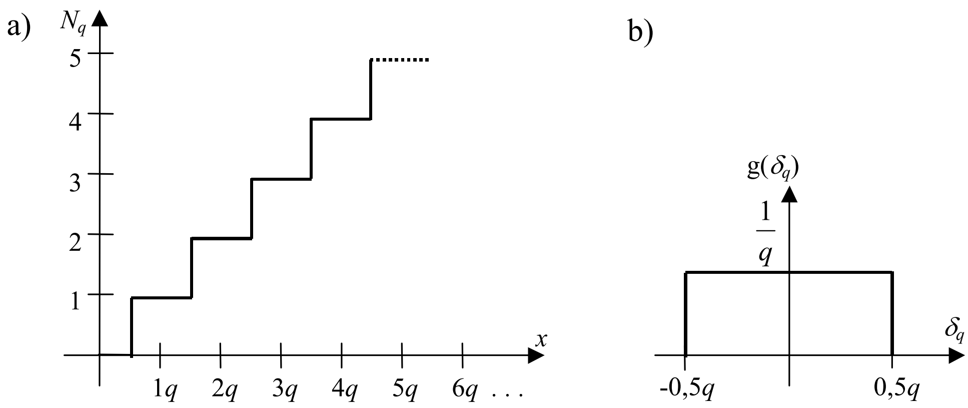


Fig. 5. a) Characteristic of the quantizer described by Eq. (11), b) distribution of its quantization error.

The definition (6) of the quantization error may be the basis of determination of the quantization result model. According to Eq. (6), one can write:

$$x = \breve{x} + \delta_q , \tag{12}$$

or for the quantizer with correction of systematic error described by Eq. (9),

$$x = \hat{x} + \delta_q \tag{13}$$

where the distribution of the error $\delta_q$ is symmetrical as shown in Fig. 5b. Both Eqs. (12) and (13) describe values of the measured quantity after quantization, i.e. the measurement result obtained in the quantization process. Accordingly with these equations, the measurement result is the sum of the quantizer indication (or the corrected quantizer indication) and a realization of the suitable error $\delta_q$. When this error is described in probabilistic categories, the quantization result has random properties. In this case, the realization of the error $\delta_q$ is taken from the set described by probability density function $g(\delta_q)$ shown properly in Fig. 4b or Fig. 5b.

The model of the quantization result described above contains only one error caused by the quantum character of the standard. In practice, there are several error sources that influence the quantization result, such as nonlinearities, noise, drifts and others. Assuming that the all separated errors influence the result additively, one can write that the general model is of the form of sum:

$$x = \hat{x} + \sum_{i=1}^{I_q} \delta_{qi} , \tag{14}$$

where $I_q$ is the number of all errors separated in the quantization process. Relations between the errors in Eq. (14) can be determined in the simulation way as described in Chapter 4.

## 3. MEASUREMENT RESULT MODEL OF SINGLE SAMPLE OF SYSTEM INPUT QUANTITY

With the assumption that all input quantities in a system are measured by using cards built as shown in Fig. 1, the scheme of the measuring chain for every quantity is of the form presented in Fig. 6. Propagation of measurement information from the input to the output of the chain can be described in signal categories. Initially, the analog input quantity $x(t)$ is converted by the sensor **S** to a voltage signal $y_S(t)$, which after that is amplified (by the element denoted as **A**) to the level suitable for next operations. The conditioned signal $y_A(t)$ is sampled by the **S/H** circuit at the moment $t_s$ and then quantized by an analog-to-digital converter **ADC**. The quantization result is delivered to the output of the **ADC** as the number of quantum assigned to the value of the sample. The input signal $x(t)$ is burdened by the disturbances denoted as $\delta_{in}(t)$ and every element of the chain introduces its own errors denoted properly as $\delta_S(t)$, $\delta_A(t)$, $\delta_{S/H}(t)$ which are treated as additive to the propagating signal. The element input signals, burdened by errors, are distinguished by a tilde.

Further considerations concern the method of building a model of a single output datum, which is the measurement result of one sample of the input quantity. Such a kind of model has to contain a description of the error sources, which influence the inaccuracy of the result, with the assumption that all systematic errors have been corrected. To simplify considerations it is assumed that the chain can be treated as linear.

For the presented assumptions, the sample at the output of the **S/H** circuit taken at the moment $t_s$ can be described as:

$$\tilde{y}(t_s) = \left[\left(\left(\left(x(t)\right) + \delta_{in}(t)\right)S_S + \delta_S(t)\right)S_A + \delta_A(t)\right]_{t=t_s} + \delta_{S/H}(t_s), \tag{15}$$

where $S_S$ is the sensitivity of the sensor **S** and $S_A$ is the amplification coefficient (sensitivity) of the amplifier **A**. Both values of $S_S$ and $S_A$ are constant.

The sample $\tilde{y}(t_s)$ is measured on the principle of quantization. It means that, basing on the quantization result model (14), the measurement result of this sample can be written as:

$$\tilde{y}(t_\mathrm{s}) = \hat{y}(t_\mathrm{s}) + \sum_{i=1}^{I_q} \delta_{qi}(t_\mathrm{s}), \tag{16}$$

where $\hat{y}(t_s)$ is the evaluation of the sample value and $\hat{y}(t_\mathrm{s}) = qN_q(t_\mathrm{s})$, $N_q(t_\mathrm{s})$ is the number (indication) at the output of the AD converter after quantization of the sample, $\delta_{qi}(t_\mathrm{s})$, $i = 1,\ldots,I_q$ are realizations of the errors of the quantization process. Comparing Eq. (15) with (16), one obtains:

$$\hat{y}(t_\mathrm{s}) + \sum_{i=1}^{I_q} \delta_{qi}(t_\mathrm{s}) = S_\mathrm{S}S_\mathrm{A}x(t_\mathrm{s}) + S_\mathrm{S}S_\mathrm{A}\delta_{in}(t_\mathrm{s}) + S_\mathrm{A}\delta_\mathrm{S}(t_\mathrm{s}) + \delta_\mathrm{A}(t_\mathrm{s}) + \delta_\mathrm{S/H}(t_\mathrm{s}). \tag{17}$$

After transformation of Eq. (17), we have:

$$x(t_\mathrm{s}) = \frac{1}{S_\mathrm{S}S_\mathrm{A}}\hat{y}(t_\mathrm{s}) + \delta_\mathrm{in}(t_\mathrm{s}) + \frac{1}{S_\mathrm{S}}\delta_\mathrm{S}(t_\mathrm{s}) + \frac{1}{S_\mathrm{S}S_\mathrm{A}}\left[\delta_\mathrm{A}(t_\mathrm{s}) + \delta_\mathrm{S/H}(t_\mathrm{s}) - \sum_{i=1}^{I_q}\delta_{qi}(t_s)\right]. \tag{18}$$

Denoting

$$c = \frac{1}{S_\mathrm{S} S_\mathrm{A}} \tag{19}$$

and the coefficients of the errors properly as $a_1, a_2, \ldots, a_I$, one can write Eq. (18) in the form:

$$x(t_\mathrm{s}) = c\,\hat{y}(t_\mathrm{s}) + a_1\delta_1(t_\mathrm{s}) + a_2\delta_2(t_\mathrm{s}) + \ldots + a_I\delta_I(t_\mathrm{s}), \tag{20}$$

where $I$ is the number of all errors separated in the described measuring chain.

Equation (20) is the mathematical model of the instantaneous value (sample) of the system input quantity $x$ at the moment $t_s$. Accordingly with the general treatment, applied at first to the model (13) of a quantization result, the sample model after measurement may be written as:

$$x(t_\mathrm{s}) = \hat{x}(t_\mathrm{s}) + \delta_x(t_\mathrm{s}), \tag{21}$$

where $\hat{x}(t_s)$ is the evaluation of the sample value and $\delta_x(t_\mathrm{s})$ is a realization of the sample resultant error. As the result of comparison of Eqs. (20) and (21), one obtains two equations. The first one is:

$$\hat{x}(t_\mathrm{s}) = c\,\hat{y}(t_\mathrm{s}) \tag{22}$$

and, when one takes into account Eq. (3), it has the form:

$$\hat{x}(t_\mathrm{s}) = c\,q\hat{N}(t_\mathrm{s}). \tag{23}$$

Equation (23) describes the dependence of the sample value from the number $\hat{N}_q(t_\mathrm{s})$ obtained at the output of the A/D converter as the result of quantization at the moment $t_s$. Therefore, the Eq. (23) may be treated as the reconstruction equation [8] of the input quantity sample.

The second equation

$$\delta_x(t_\mathrm{s}) = a_1\delta_1(t_\mathrm{s}) + a_2\delta_2(t_\mathrm{s}) + \ldots + a_I\delta_I(t_\mathrm{s}), \tag{24}$$

describes relations between the value of the sample resultant error and realizations of partial errors arise during signal processing in the measuring chain. Equation (24) determines the error model of the input quantity sample measured in the system by using the described chain. It is a linear combination of the sample partial errors and constant coefficients $a_1, a_2, \ldots, a_I$.

## 4. IDENTIFICATION AND VERIFICATION OF THE ERROR MODEL

Identification of the model given by Eqs. (23) and (24) consists in determination of the coefficients and descriptions of errors. As it results from Eq. (17), values of the coefficients can be determined on the basis of characteristics of the chain elements or of measured values, whereas the problem of error description is much more sophisticated because the errors depend on working conditions of the measuring chain. To simplify considerations, let us assume that the input quantity is varying in time periodically and one instantaneous value (sample) of it is measured. The measurement moment $t_s$ is not correlated with the phase of the input quantity waveform, which means that the sampling moment may be described as random in this quantity period.

Identification of the error sources consists in searching for a function describing sets of the error values arisen in working conditions and determining relations between the errors if such relations exist. The simplest way is to find suitable parameters in the materials delivered by the producers of elements, for example the noise variance of the amplifier applied in the chain. Some parameters may be determined analytically on the basis of signal processing analysis as it has been done in Chapter 2 for the quantization error, the distribution of which is shown in Fig. 5b. But one should notice that these possibilities are seldom applied in practice and the main way of identification must be realized experimentally, especially when the error is described in probabilistic categories.

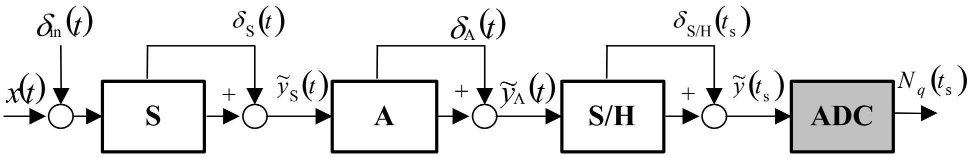


Fig. 6. Propagation of the input signal in the system measuring chain.

There are two kinds of experiments useful in this situation – performed by means of measurements (physically) or in a simulation way with the use of the Monte Carlo method. Both kinds consist in acquisition of values of the selected error in the determined measurement conditions. Obtained in such a way, the error value set is then presented in histogram form which, if possible, can be approximated by a probability density function. The relation between a pair of error sources is described by a correlation coefficient [6].

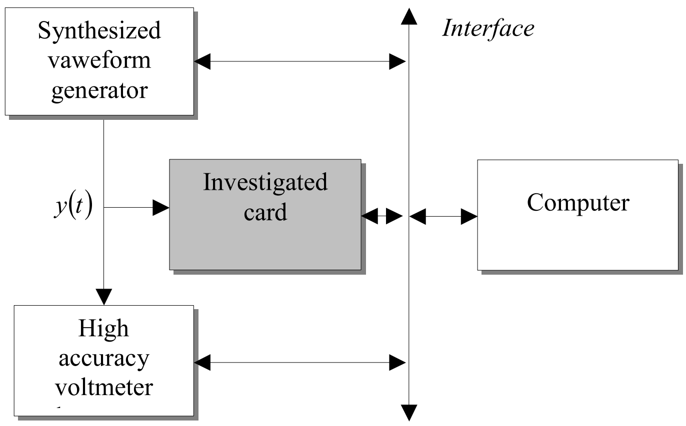


Fig. 7. System for identification of the measuring card errors.

System elements, errors of which must be determined first of all, are measuring cards. The scheme of an experimental system for identification error sources of a measuring card is shown in Fig. 7. The input of the investigated card is connected with a generator which produces digitally synthesized voltage waves calibrated by using a high accuracy digital voltmeter. Fig. 8 describes the identification procedure. One step of the procedure consists in measuring a sample of the input signal at the selected moment $t_s$ and comparing the obtained result $\tilde{y}(t_s)$ with the known instantaneous value $y(t_s)$ of the synthesized signal. Subtraction of the compared values gives one error value $\delta(t_s)$ which is introduced to the error value set represented by a histogram.

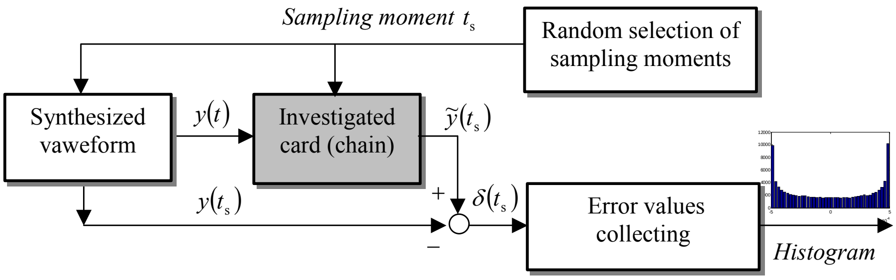


Fig. 8. Scheme of procedure of error value set determination.

The identification procedure needs a great number of steps to obtain one histogram and, moreover, experiments have to contain many procedures performed in such measuring conditions which enable to extract a description of one error source from histograms containing values of several partial errors. Problems which appear in practice while using the measurement method have been described in [5] where an experimental system applying a VME bus standard and exemplary histograms of errors obtained for the VADC32 card have been presented.

The measurement identification of error sources is a time-consuming process and needs high quality measurement tools organized in a system. For some errors, the same effect can be achieved in a much simpler way – by using simulation. The scheme of obtaining an error value set is the same as shown in Fig. 8 but the whole procedure is realized using simulated data. Properties of this kind of identification procedure are illustrated by the following example.

**Example**

Let us take into consideration the measuring chain shown in Fig. 9 with the assumption that only two errors are taken into account. The first one is the dynamic error introduced by the amplifier **A** while the second one is connected with the quantization process performed by the analog-to-digital converter **ADC**.

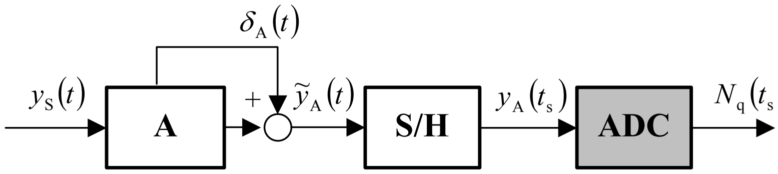


Fig. 9. Scheme of exemplary measuring chain.

The aim of the described simulation experiment is to verify the hypothesis that these two errors mentioned above may be composed according to the general linear Eq. (24) in the situation when the quantization process is nonlinear, as described by Eq. (11). To investigate this problem let us assume that the input signal of amplifier **A** is sinusoidal, i.e. $y_S(t) = Y_S \sin \omega t$, $\omega = 2\pi f$, where $f$ is the frequency and $Y_S$ is the amplitude of the signal. Let the dynamic properties of the amplifier be described by a first-order linear differential equation. Taking into account these assumptions, the amplifier spectral transmittance is of the form:

$$S_A(j\omega) = \frac{\tilde{Y}_A(j\omega)}{Y_S(j\omega)} = \frac{S_A}{1 + j\dfrac{\omega}{\omega_0}} = \frac{S_A}{1 + j\dfrac{f}{f_0}}, \tag{25}$$

where $f_0$ is the cut-off frequency (bandwidth) and $S_A$ - the amplification coefficient (static sensitivity of the amplifier).

Accordingly with Eq. (25), the time form of the amplifier output signal is given as:

$$\tilde{y}_A(t) = Y_A \sin(\omega t + \varphi), \tag{26}$$

where:

$$Y_A = \left|Y_S(j\omega) S_A(j\omega)\right| = \frac{Y_S S_A}{\sqrt{1 + \left(\frac{f}{f_0}\right)^2}} \tag{27}$$

and

$$\varphi = -\arctan\left(\frac{f}{f_0}\right). \tag{28}$$

The dynamic error of a measuring transducer is generally defined as:

$$\delta(t) = y(t) - y_{\text{ideal}}(t), \tag{29}$$

where $y_{\text{ideal}}(t)$ is the waveform at the output of the transducer which is treated as dynamically ideal. The transmittance of the ideal transducer does not depend upon frequency, i.e. it is equal to the amplification coefficient $S_A$ when the amplifier is taken into account. From Equations (25) and (29) it results that the spectral form of the amplifier dynamic error in the considered conditions is given as:

$$\begin{aligned} \delta_{\text{dyn}}(j\omega) &= Y(j\omega) - Y_{\text{ideal}}(j\omega) = Y_S(j\omega)\left[S_A(j\omega) - S_A\right] = \\ &= Y_S\left(\frac{S_A}{1 + j\frac{f}{f_0}} - S_A\right) = Y_S S_A \frac{-j\frac{f}{f_0}}{1 + j\frac{f}{f_0}} \end{aligned}. \tag{30}$$

Therefore, the amplitude of the dynamic error is described by the expression:

$$E_{\text{dyn}} = \left|\delta_{\text{dyn}}(j\omega)\right| = Y_S S_A \frac{\frac{f}{f_0}}{\sqrt{1 + \left(\frac{f}{f_0}\right)^2}}. \tag{31}$$

To simplify considerations, let us take that the amplitude $Y_S$ of the input signal is equal to 1V and the amplification coefficient $S_A = 1\frac{\text{V}}{\text{V}}$. Moreover, it is assumed that the cut-off frequency is $f_0 = 10^5$ Hz and the frequency of the input signal is $f = 50$ Hz. In this case, it follows from Eq. (31) that the amplitude of the amplifier dynamic error is:

$$E_{\mathrm{dyn}} \cong Y_{\mathrm{S}} S_{\mathrm{A}} \frac{f}{f_0} = 1\,\mathrm{V} \frac{\mathrm{V}}{\mathrm{V}} \frac{50\,\mathrm{Hz}}{10^5\,\mathrm{Hz}} = 5 \cdot 10^{-4}\,\mathrm{V}. \tag{32}$$

The amplifier output signal, being the sum of the ideal one and the dynamic error, is sampled at the moment $t_s$ and then quantized. Let us take that quantization is performed by a 12-bit, bipolar, binary AD converter, the input range of which is from –1V to 1V. Therefore, the value of its quantum is:

$$q = \frac{1\,\mathrm{V} - (-1\,\mathrm{V})}{2^{12}} = 2^{-11}\,\mathrm{V}. \tag{33}$$

The aim of the simulation experiment is the determination of the histogram which describes the resultant error value set being a composition of the dynamic and quantization errors and then calculation of the resultant variance. The experiment has been realized in 200 000 steps. Every step consists of several activities and it begins with the selection of the sampling moment $t_{\mathrm{s}}$. Taking into account that sampling may be initiated at any moment, one assumes that it is determined randomly with rectangular distribution in the period of the input signal, the value of which is:

$$T = \frac{1}{f} = \frac{1}{50}\,\mathrm{s} = 0.02\,\mathrm{s}. \tag{34}$$

Next, using Eq. (26), the value of one sample has been calculated and the obtained number has been located at the suitable bar of the dynamic error histogram. Moreover, the sample has been quantized accordingly with Eq. (11) and the quantization result has been placed at the second histogram and used for calculation of the resultant error variance.

The histograms, obtained as the results of the described experiment, are shown in Fig. 10.

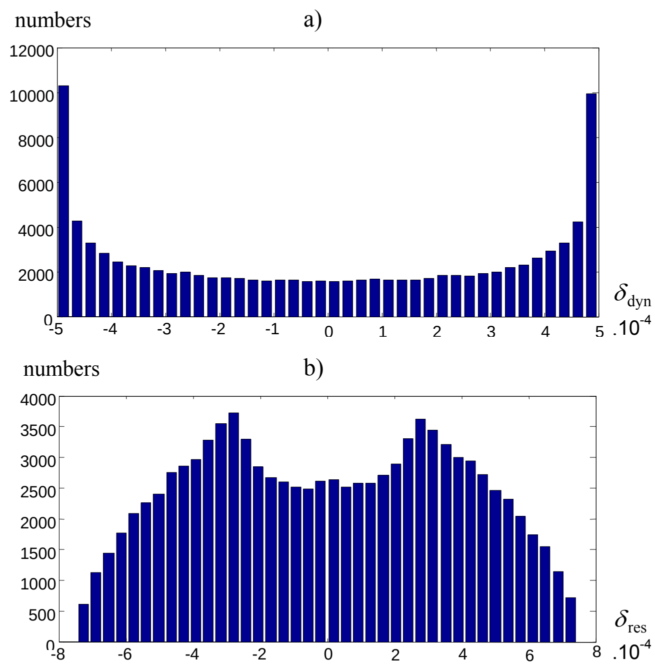


Fig. 10. Exemplary histograms: a) amplifier dynamic error, b) resultant error composed of dynamic and quantization error.

On the basis of the resultant error variance calculated in the experiment, one can determine the standard deviation of this error. It takes the value:

$$\sigma_{\mathrm{exp}} = 3.8096 \cdot 10^{-4}\ \mathrm{V}. \tag{35}$$

This value can be compared with the value of the standard deviation obtained as the result of theoretical considerations, which enables to arbitrate whether any correlation between the dynamic and the quantization error exists what can be expected as the quantization process is nonlinear (see Eq. (11)). The value of the standard deviation of a sinusoidal signal with amplitude $A$ is $A/\sqrt{2}$ and the standard deviation of the quantization error with rectangular distribution, shown in Fig. 5b, is $q/2\sqrt{2}$ [6]. Therefore, basing on the values given by Eqs. (32) and (33), one can determine analytically the resultant error standard deviation value as [9]:

$$\sigma_{\mathrm{anal}} = \sqrt{\sigma_{\mathrm{dyn}}^2 + \sigma_{\mathrm{q}}^2} = \sqrt{\frac{E_{\mathrm{dyn}}^2}{2} + \frac{q^2}{12}} = 3.8062 \cdot 10^{-4}\ \mathrm{V}. \tag{36}$$

Comparing the values given by Eqs. (35) and (36), one can draw the conclusion that nonlinearity of the quantization has no influence on relations between the dynamic and quantization error because its correlation coefficient is practically equal to 0 [6]. To make sure if this statement is true for a large enough range of quantum values, one has determined the standard deviation values in the simulation and analytical way for typical bit numbers of AD converters. The obtained values are presented in Table 1.

Table 1. Experimentally $\sigma_{\mathrm{exp}}$ and analytically $\sigma_{\mathrm{anal}}$ determined values of the dynamic and quantization error standard deviations obtained for $n$-bit AD converters with assumption that amplitude of the dynamic error is equal to the maximum value of the quantization error.

| $n$ | 4 | 8 | 12 | 16 |
|---|---|---|---|---|
| $\sigma_{\mathrm{exp}}$ [V] | $5.62 \cdot 10^{-2}$ | $3.60 \cdot 10^{-3}$ | $2.22 \cdot 10^{-4}$ | $1.39 \cdot 10^{-5}$ |
| $\sigma_{\mathrm{anal}}$ [V] | $5.70 \cdot 10^{-2}$ | $3.62 \cdot 10^{-3}$ | $2.23 \cdot 10^{-4}$ | $1.39 \cdot 10^{-5}$ |

## 5. FINAL REMARKS AND CONCLUSIONS

The model of a measurement result presented in the paper has been developed on the basis of an analysis of the quantization process and description of properties of the typical measuring chain used in systems. It is necessary to emphasize that it is **the model of one measurement result**. There is no problem to use the model in the situation when one calculates the mean value of a series of measurement results obtained in the same conditions (for a constant input signal). Generally, one can state that the described model can be applied as the basis of an error model construction for practically all types of data processing algorithms used in measuring systems [2]. It permits to build a system error model.

In accordance with the presented model, the measurement result is the sum of the result evaluation and the realization of the resultant error described as a linear combination of partial errors. It means that descriptions of all these errors have to be known before using the model for calculation of the result's inaccuracy. The error identification can be performed both in an analytical and experimental way, in the second case one can do it by using measurement or simulation experiments.

For a time-varying input quantity and in a situation when the moment of sampling this quantity is unrestricted (practically admitted as random), the partial errors are described in probabilistic categories. Ina such a case, inaccuracy of a measurement result can be expressed as uncertainty [9]. The definition which enables defining uncertainty as a parameter of an error value set has been proposed in [2, 3]. Knowing the error model of the measuring system and basing on this definition, one may use the mathematical apparatus described in paper [2] for self-determination of uncertainties of data obtained at the system output.

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