

ZDZISŁAW KACZMAREK, MAŁGORZATA DETKA

Kielce University of Technology
Division of Theoretical Electrical Engineering and Metrology
Poland, e-mail: m.detka@tu.kielce.pl

THE RECONSTRUCTION OF WAVEFORMS OF IMPULSIVE STRAIN OCCURRING IN AN ELASTIC BAR BY MEANS OF DECONVOLUTION IN THE FREQUENCY DOMAIN

The paper presents a method of reconstruction of impulsive strain waveforms based on deconvolution in the frequency domain and the spectral transmittance of the bar, which was determined experimentally and given in an analytic form. A regularization of the deconvolution is obtained by means of limiting the frequency band of the inversion filter. In this method the one-point or the two-point measurement of strain waves in an elastic bar can be utilized.

Keywords: reconstruction, deconvolution, frequency domain, impulsive strain

1. INTRODUCTION

There exist several methods to reconstruct a measurand or rather its best estimate. One of them is the transformation method used for the reconstruction of waveforms in the frequency domain, which is based on the Fourier transform. It is employed for transducers described with the convolution-type integral equation. The reconstruction process needs to be regularized, and this involves shaping the amplitude-frequency characteristic of the reconstruction path by means of regularization filters. In Refs. [1–4] the method was used to reconstruct impulse force and pressure waveforms by employing an elastic bar to act as a mechanical transducer. It was reported that to assure adequate pulse signal reconstruction it was essential to mount strain gauge sensors at some pre-determined points along the bar. It was then possible to calculate the spectral transmittance of a bar section using the signals from the strain gauge sensors, and this transmittance was directly used to reconstruct impact waveforms. The spectral transmittance of a selected bar section was calculated in each reconstruction process.

This paper presents a method for reconstruction of impulsive strain waveforms, whose essence consists in deconvolution in the frequency domain with the use of the spectral transmittance of the elastic bar determined experimentally as an analytical relation. The method bases on the one-point or two-point measurement of strain waves

in the bar. The reconstruction process is regularized by limiting the frequency band of the inversion filter [1].

This method was used to reconstruct impulsive strain waveforms generated by longitudinal, elastic impact into a steel bar of steel balls with different diameters.

2. METHOD

The method for the reconstruction of impulsive strain waveforms using the spectral transmittance of a bar section determined experimentally is based on the one-point or the two-point measurements of strain waves propagating along the bar [1, 4]. The Lagrangian diagram in Fig. 1 illustrates the propagation of elastic strain waves in the bar and the waveforms of these waves obtained at the points of measurement. In the reconstruction process with the use of the two-point measurement of the strain waves, for the analysis, the waveforms of the incident wave at points a and b of the bar are utilized. In the reconstruction with the one-point measurement, the waveforms of the incident wave and the first reflected wave at point a are employed.

The responses of the one-point sensor, generated by a longitudinal impact into a steel bar 1.5 m long and 22 mm in diameter of steel balls of different diameters, are shown in Figs. 2 and 3.

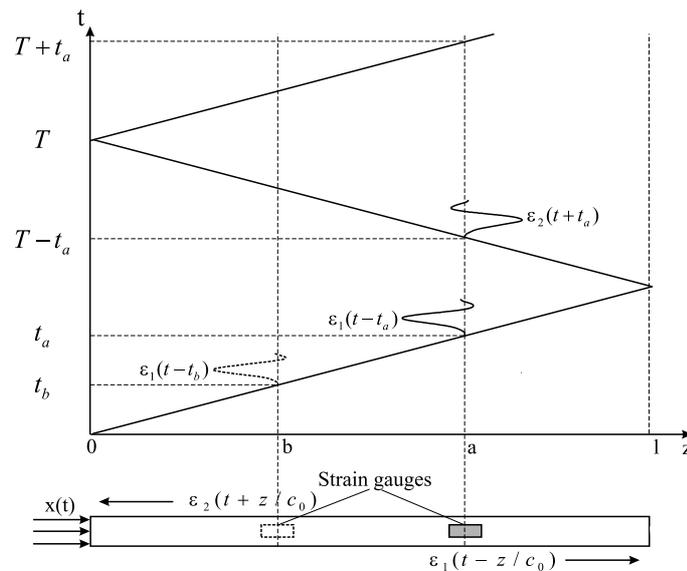


Fig. 1. Lagrangian diagram for longitudinal waves in a cylindrical bar of length l .

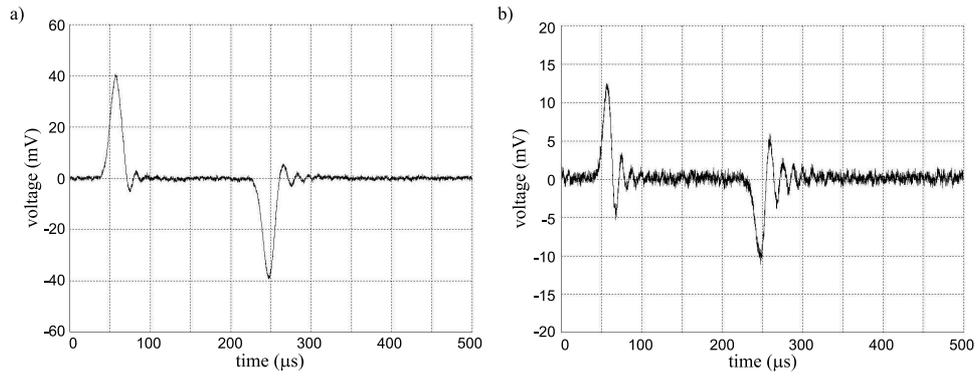


Fig. 2. Responses of the one-point sensor generated by a longitudinal impact into a steel bar of steel balls of diameters: a) 10.000 mm, b) 5.556 mm. The bar length and diameter are equal to 1.5 m and 22 mm respectively. The impact velocity $v = 2.2$ m/s.

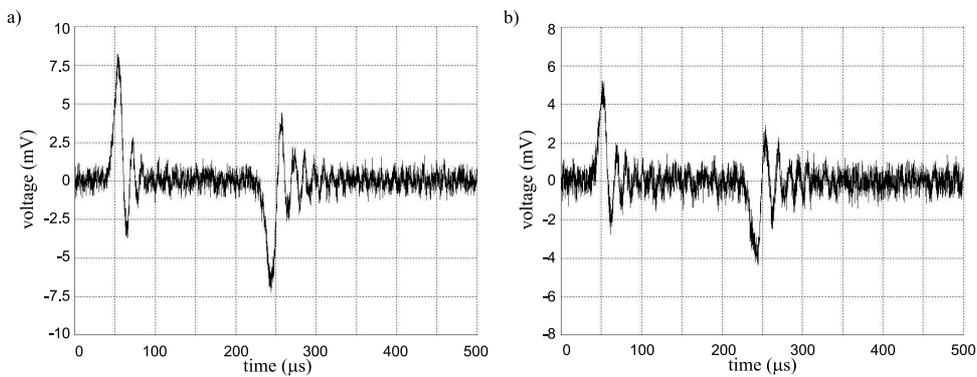


Fig. 3. Responses of the one-point sensor generated by a longitudinal impact into a steel bar of steel balls of diameters of: a) 4.762 mm, b) 3.000 mm. The bar length and diameter are equal to 1.5 m and 22 mm respectively. The impact velocity $v = 2.2$ m/s.

If the spectral function of the primary strain waveform $\varepsilon_{1a}(t)$ and the inverted first reflected strain waveform $\varepsilon_{2a}(t)$, are noted respectively as $\hat{\varepsilon}_{1a}(j\omega)$ and $\hat{\varepsilon}_{2a}(j\omega)$, then the spectral transmittance of the bar section $2(L-a) = 1$ m is [5]:

$$G_{2(L-a)}(j\omega) = \frac{\hat{\varepsilon}_{2a}(j\omega)}{\hat{\varepsilon}_{1a}(j\omega)}. \quad (1)$$

Since the length of the section in the applied sensor is $2(L-a) = 1$ m, the spectral transmittance derived from relationship (1) is the spectral transmittance of a unit section. Thus, the spectral transmittance denoted by $G(j\omega)$ can be written as:

$$G(j\omega) = G(\omega)e^{j\varphi(\omega)} = \frac{\hat{\varepsilon}_{2a}(\omega)}{\hat{\varepsilon}_{1a}(\omega)}e^{j\varphi(\omega)}, \quad (2)$$

where:

$$\varphi(\omega) = \arg[\hat{\varepsilon}_{2a}(j\omega)] - \arg[\hat{\varepsilon}_{1a}(j\omega)] = \varphi_2(\omega) - \varphi_1(\omega). \quad (3)$$

The calculated amplitude and phase spectra of the primary and the first reflected strain waves, which were excited by the longitudinal, elastic impact into a steel bar of a steel ball of 3 mm diameter, are shown in Fig. 4. On the basis of these frequency characteristics and the relationships (2) and (3) the magnitude and the argument of the spectral transmittance of a bar section with a unit length were calculated and presented in Fig. 5.

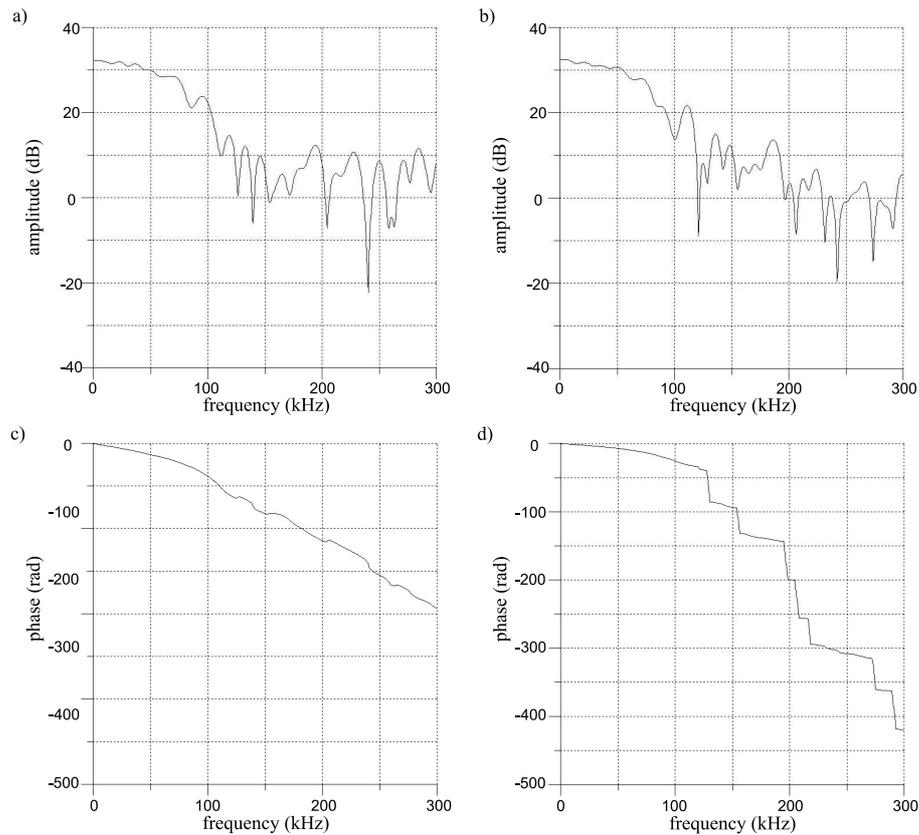


Fig. 4. Spectra of the strain waves excited by the longitudinal, elastic impact into a steel bar of a steel ball of 3 mm diameter: a) amplitude spectrum of the primary wave, b) amplitude spectrum of the first reflected wave, c) phase spectrum of the primary wave, d) phase spectrum of the first reflected wave.

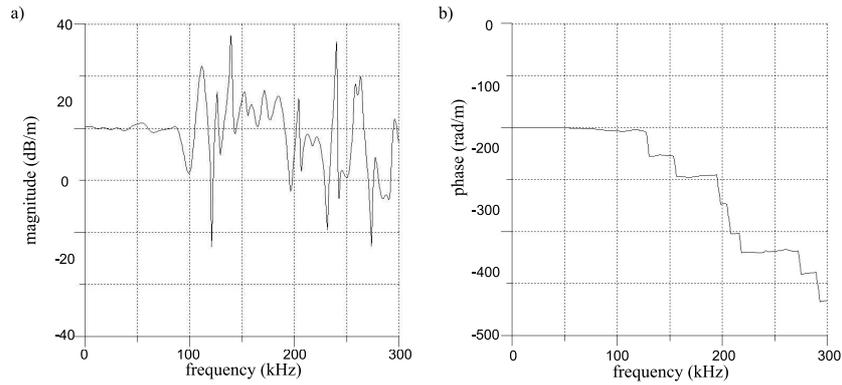


Fig. 5. Spectral transmittance of a unit bar section: a) magnitude, b) argument.

The magnitude of the spectral transmittance of a unit section of the bar was determined on the basis of numerous measurements of the strain waves caused by the 3 mm ball. As a result of mean square approximation this magnitude can be given analytically as:

$$G(\omega) = e^{-\alpha\omega^2}, \quad (4)$$

where the coefficient $\alpha = 1.56 \cdot 10^{-13} \text{ [s}^2/\text{m]}$.

The approximation results are shown in Fig. 6.

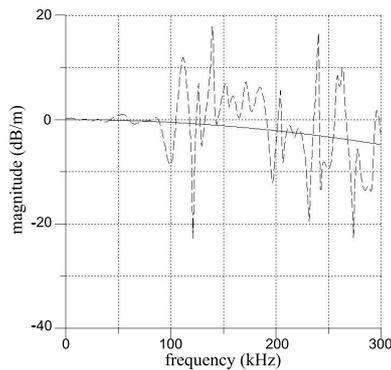


Fig. 6. Approximation of the spectral transmittance magnitude of a unit bar section.

The argument of the transmittance of a unit section of the bar was also determined on the basis of numerous measurements of the strain waves caused by the 3 mm ball. The mean square approximated waveform of this argument, together with the waveform of the phase angle φ calculated from the numerical solution of the Pochhammer-Chree

equation, is shown in Fig. 7. The phase delay φ of each harmonic component over the propagation distance of 1m in the bar can be calculated from the relation:

$$\varphi = \omega \left(\frac{1}{c_n} - \frac{1}{c_0} \right), \quad (5)$$

where:

c_0 – velocity of longitudinal waves of an infinite length,

c_n – phase velocity of each frequency component determined from the dispersion curve $c_n/c_0 = f(\alpha/\lambda)$ for $\nu = 0.3$ [6].

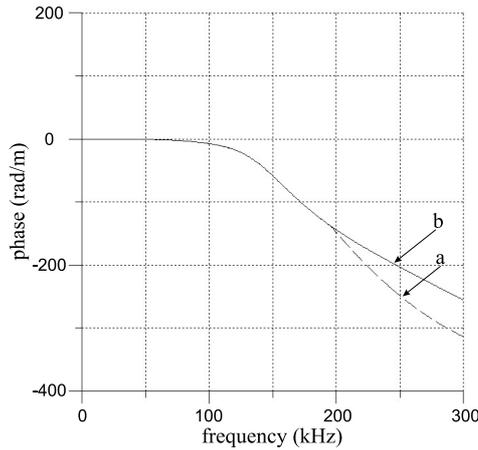


Fig. 7. Spectral transmittance argument determined: a) experimentally, b) on the basis of the theoretical analysis.

Thus, the spectral transmittance of a one-meter-long unit section of the bar used in the experiments can be described by the formula:

$$G(j\omega) = e^{-\alpha\omega^2 - j\varphi}. \quad (6)$$

Taking into account in relationship (6) the general character of the damping function and the linearity of the phase delay φ in relation to length l , one gets the following form of the spectral transmittance of an arbitrarily long bar section, length l being expressed in meters [5]:

$$G^l(j\omega) = e^{-\alpha l\omega^2 - jl\varphi}. \quad (7)$$

The strain wave spectrum estimate $\tilde{\varepsilon}_0(j\omega)$ is reconstructed using the following formula:

$$\tilde{\varepsilon}_0(j\omega) = G^{-l}(j\omega)\hat{\varepsilon}_l(j\omega), \quad (8)$$

where $\tilde{\varepsilon}_0(j\omega)$, $\hat{\varepsilon}_l(j\omega)$ denote the Fourier transforms of the estimate of the strain waveform for $z = 0$ and of the strain waveform recorded at distance $z = l$ from the impact end of the bar.

The strain waveform estimate $\tilde{\varepsilon}_0(t)$ is determined from the relationship:

$$\tilde{\varepsilon}_0(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{\varepsilon}_0(j\omega) e^{j\omega t} d\omega. \quad (9)$$

3. RECONSTRUCTION RESULTS

The proposed method for the reconstruction of impulsive waveforms, using the experimentally determined spectral transmittances of the bar, was applied to reconstruct impulsive strain waveforms generated by the impact of steel balls with diameters of 10.000 mm, 5.556 mm, 4.762 mm and 3.000mm. The reconstruction was conducted in accordance with relationships 8 and 9 using the strain waveform ε_{1a} measured at point a . The results of the reconstructed strain waveform estimates for the particular balls are presented in Figs. 8–11.

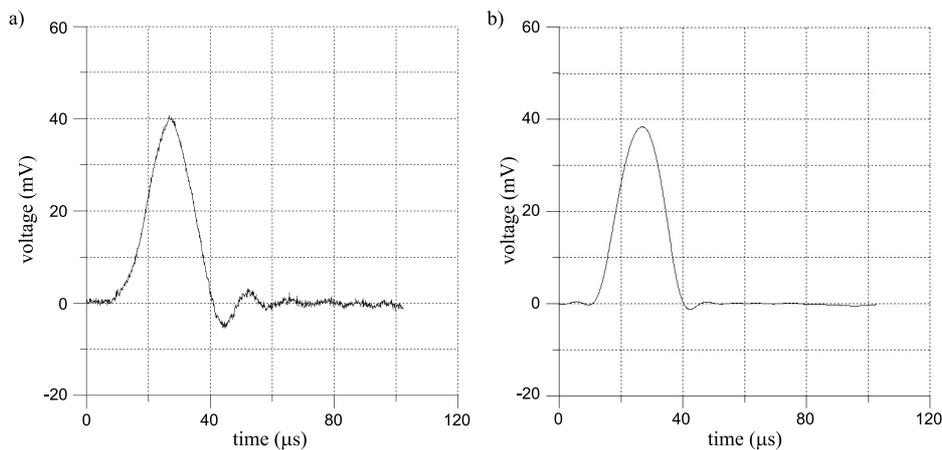


Fig. 8. Strain waveform excited by a longitudinal impact into a steel bar of a steel ball of 10.000 mm diameter: a) the measured incident pulse, b) the reconstructed incident pulse.

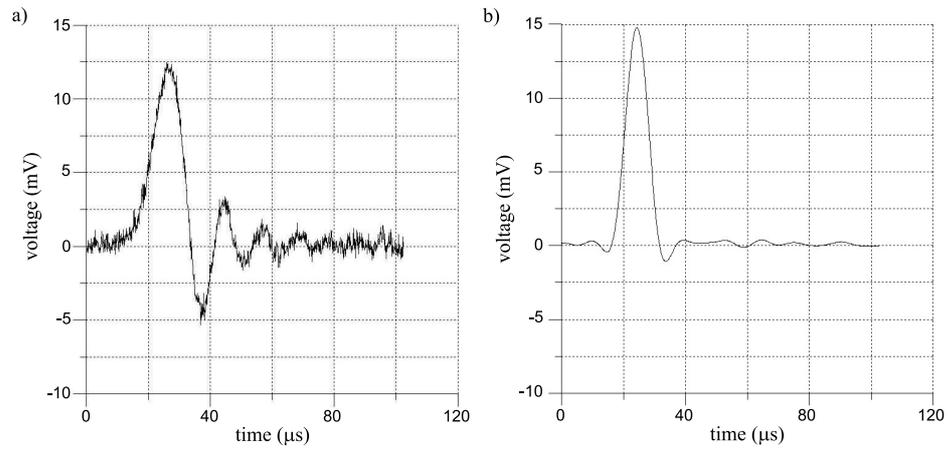


Fig. 9. Strain waveform excited by a longitudinal impact into a steel bar of a steel ball of 5.556 mm diameter: a) the measured incident pulse, b) the reconstructed incident pulse.

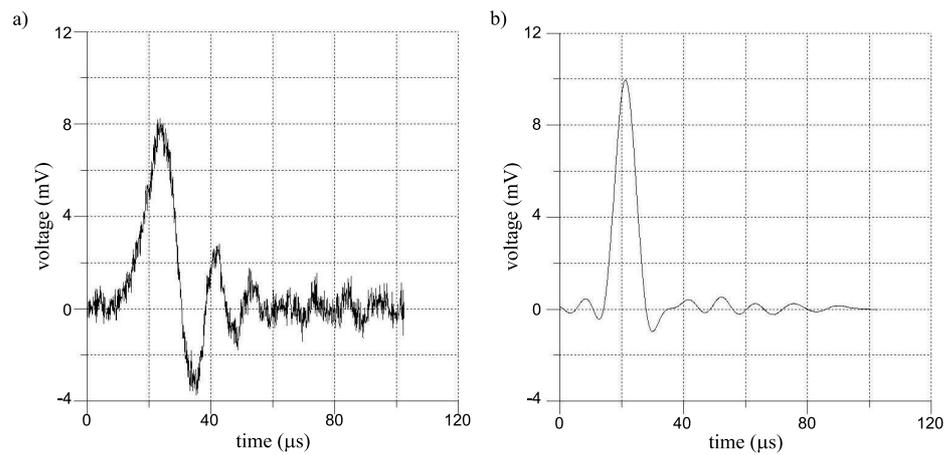


Fig. 10. Strain waveform excited by a longitudinal impact into a steel bar of a steel ball of 4.762 mm diameter: a) the measured incident pulse, b) the reconstructed incident pulse.

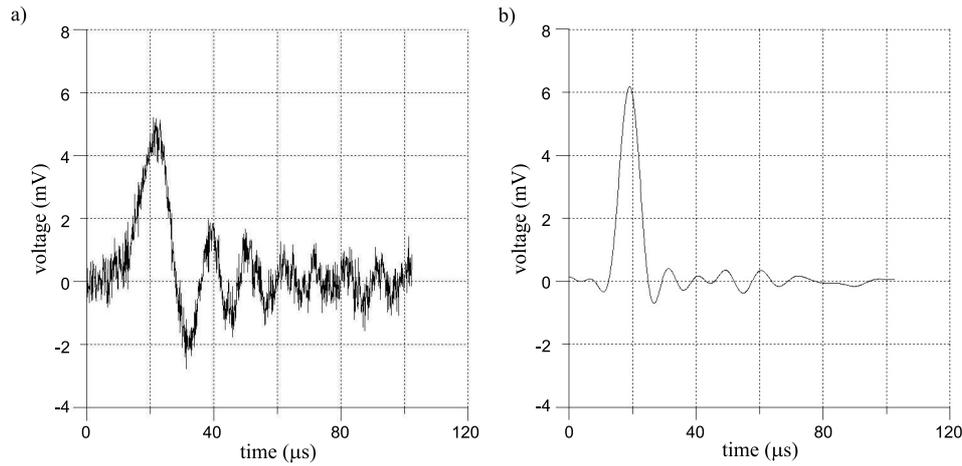


Fig. 11. Strain waveform excited by a longitudinal impact into a steel bar of a steel ball of 3.000 mm diameter: a) the measured incident pulse, b) the reconstructed incident pulse.

4. EXPERIMENTAL VERIFICATION OF THE METHOD

The method's correctness was verified experimentally by comparing the measured waveforms of the first reflected strain wave with their estimates calculated on the basis of the measured waveforms of the direct strain wave and the determined spectral transmittance of the bar. The measured waveforms of the first reflected strain wave and their calculated estimates are shown in Figs. 12 and 13. The quantitative comparison for the main impulse of the waveforms was performed by assessing the coincidence of amplitude A and duration a of the measured waveforms and amplitude \tilde{A} and duration \tilde{a} of the waveforms of the calculated estimates. The coincidence of the main impulses of the measured waveforms and their calculated estimates was determined on the basis of the values of the relative amplitude error and the relative duration error. The assessment results given in Tables 1 and 2 confirm the method's correctness as well as the correctness of the spectral transmittance of the bar.

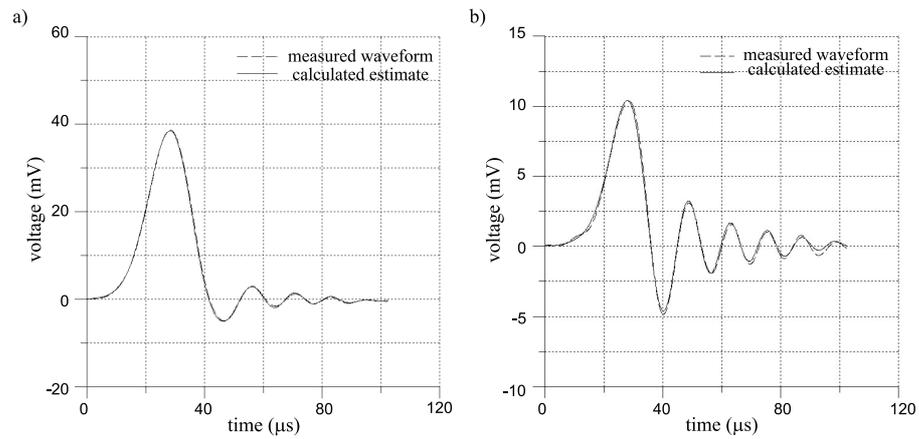


Fig. 12. Measured and estimated waveforms of the first reflected strain wave generated in the bar by a longitudinal impact into it of steel balls with diameters of a) 10.000 mm, b) 5.556 mm.

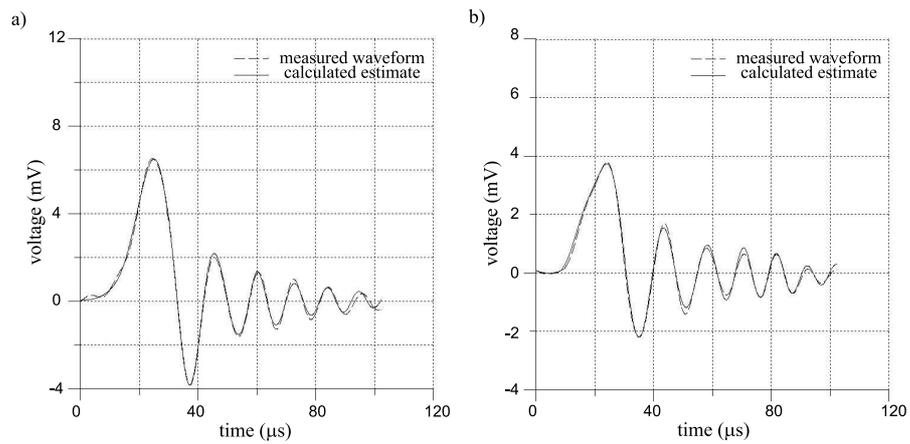


Fig. 13. Measured and estimated waveforms of the first reflected strain wave generated in the bar by a longitudinal impact into it of steel balls with diameters of: a) 4.762 mm, b) 3.000 mm.

Table 1. Relative error in the estimation of the main pulse duration.

Sphere diameter	$\tilde{\alpha}$	α	$\frac{\tilde{\alpha} - \alpha}{\alpha}$
mm	μs	μs	%
10.000	41.4	41.6	0.4
5.556	35.8	35.9	0.2
4.762	32.7	32.9	0.6
3.000	23.7	24.6	3

Table 2. Relative error in the estimation of the main pulse amplitude.

Sphere diameter	\tilde{A}	A	$\frac{\tilde{A} - A}{A}$
mm	mV	mV	%
10.000	38.51	38.67	0.4
5.556	10.41	10.38	0.2
4.762	6.50	6.48	0.3
3.000	3.73	3.78	1.3

5. CONCLUSIONS

The spectral transmittance of the elastic bar determined from measurements, given in the form of an analytical relation permits the reconstruction and simulation of a strain waveform in any of its cross-sections, and, in the instance of a measured waveform, in one of its cross-sections. The reconstruction of the strain waveform is performed as deconvolution in the frequency domain. The regularization of the deconvolution is done by limiting the band of the inversion filter. The results of the experimental verification confirm that the method is correct. The method can be used to reconstruct impulsive strain waveforms when using an elastic bar as a mechanical transducer within its operation range with dispersion.

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