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SPECIFIC MASS FLOW RATE MEASUREMENTS IN A PULSATING FLOW OF GAS

The method of measurement of the specific mass flow rate in a pulsating flow by means of the Constant Temperature Anemometer (CTA) has been described. Special attention has been paid to the probe calibration problem. Different forms of the CTA characteristic have been analyzed. Example measurement results have been presented for chosen pulse frequencies. Apart from the measurements executed in one single point representing each control section, a flow field survey has been done in order to determine the velocity profile under the conditions of a pulsating flow. Probes have been displaced radially with a small step to cover the range from the pipe axis to its wall. It has been found that in the large field around the pipe axis, successive velocity plots are similar as far as their shape and phase shift are considered. Pulsations are damped and mean velocity decreases rapidly only in direct proximity of the pipe wall. It has been also shown that the presence of pulsations makes the mean velocity profile more uniform than in the case of a steady flow. Measurements have been also performed for the case of a reverse flow occurring in the pipe at resonance frequencies.

Keywords: pulsating flow, hot wire probe, calibration, approximation, flow field survey

1. INTRODUCTION

In case of an unsteady flow of gas, the main parameters describing the state of the working medium are time-dependent functions. In the particular case of a pulsating flow these functions become periodical.

The pulsating flow is usually met in inlet and exhaust pipes of internal combustion engines and piston compressors. Its unsteadiness results from cyclic opening and closing of inlet or exhaust valves. For the fixed operation point of the engine or the compressor, the pulse frequency is constant and depends on the engine current rotational speed, whereas for multi-cylinder engines – on the number of cylinders and on the engine operation principle (two or four-stroke engine).

Usually three parameters are used to describe the flow in pipes connected to the engine, i.e., pressure, temperature and velocity, related to volumetric or mass flow rate, which will be of particular interest in this article.

Contrary to other parameters, which are scalars, velocity is a vector quantity:

$$\vec{v} = \vec{v}(x, y, z, t)\,. \tag{1}$$

Its components in the Cartesian system of coordinates x, y, z can be denoted as v_x, v_y, v_z, respectively.

In case of the flow through pipes, the axial velocity component, which is directed along the pipe axis, is taken into account. This component is referred to as the mean velocity or the mass transport velocity, because it is the result of transport of a certain quantity of the fluid through the analyzed control section. For balance purposes, either the space-averaged volumetric flow rate $\hat{V}$:

$$\hat{V} = \hat{v}_x S_p, \tag{2}$$

where: $\vec{v}$ – axial velocity component averaged through the control section surface; S_p – control section surface; or the averaged mass flow rate $\hat{m}$[1]:

$$\hat{m} = \rho\hat{V} = \rho\hat{v}_x S_p, \tag{3}$$

where: ρ – averaged flow density for the control section in question, is used.

In the case of pulsating flow analyzed, the transient values of flow parameters should be taken into account to correctly describe the flow dynamics and unsteady phenomena.

For the unsteady flow $\left(\widehat{v}_x = \widehat{v}_x(t)\right)$, both flow rates (2) and (3) become time-dependent functions: $\hat{V} = \hat{V}(t)$ and $\hat{m} = \hat{m}(t)$.

Such an approach, which employs averaged quantities, allows us to use only one axial velocity for the whole control section, without applying the function or the map describing its space distribution in this section. The temporal variation of this velocity is maintained in accordance with the flow character.

It is also possible to use the one-dimensional flow theory [2], applying certain correction coefficients if needed, which take into account the influence of the boundary layer and other factors affecting the velocity profile shape in the control section under analysis.

In order to simplify the notations, the symbol v will be used for the axial component of velocity in the further part of this paper.

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[1] PN-ISO 31-3: 2000 specifies the notation for volumetric flow rate as q, and for mass flow rate as q_m. Nevertheless author applies traditional notations ($\hat{V}$ and $\hat{m}$) which are still in common use [1].

2. APPLICATION OF HOT WIRE PROBES FOR VELOCITY MEASUREMENTS UNDER THE CONDITIONS OF A PULSATING FLOW OF GAS

Hot wire probes with tungsten wires, working in the CTA (Constant Temperature Anemometer) mode, were used in the described investigations on a pulsating flow. This choice was imposed by their metrological qualities, i.e., excellent dynamic properties, small dimensions of the measuring tip[2], which allows the measurement to be considered as punctual, as well as acceptable costs of purchase and exploitation. As an alternative solution, only laser optical techniques can be considered, however their application is still very expensive.

According to Bruun [3], the frequency limit for such anemometers can be estimated as 1×10^5 Hz. Because maximal excitation frequencies generated during the presented tests attained the value of 200 Hz[3], the applied probes can be considered as inertialess in this range of frequencies, even if a few first harmonics are taken into account.

2.1. Choice of the conversion equation

On the basis of the equation of heat balance for a thin wire [4, 5], a relation between the output voltage signal U and the flow velocity v in the wire neighbourhood[4] can be obtained:

$$U^2 = A' + B' (\rho v)^N \left(T_w - T_g\right), \tag{4}$$

where: A',B',N – coefficients to be determined during calibration; T_w – wire temperature; T_g – gas temperature; ρ – gas density.

As can be seen from Eq. (4), the hot-wire output signal depends not only on the gas velocity v, but also on its density ρ which is also variable under the conditions of a pulsating flow. Therefore, the specific mass flow rate φ_m should be defined as:

$$\varphi_m = \rho v = \frac{\dot{m}}{S_p}, \tag{5}$$

which is in fact the instantaneous mass flow rate $\dot{m}$ related to the pipe cross-section area S_p.

In case of the application of the Constant Temperature Anemometer (CTA), the wire temperature T_w is kept constant by the electronic control system. If calibration of

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[2] The diameter of applied wires was $d_w = 5\mu$m and their length $l_w = 2$ mm. For the pipe diameter $d = 42$ mm, respective geometric ratios are: $dw/d = 1.2 \times 10^{-4}$ and $l_w/d = 0.048$.

[3] This value corresponds to the pulse frequency of exhaust gases for the 4-stroke, 4-cylinder engine operating at the maximal rotational speed $n_s = 6000$ rev/min.

[4] In this paper, gas flow is considered, therefore all the equations and the presented analysis correspond to this case.

the wire is conducted at the same temperature of gas T_g as the mean flow temperature during measurements in the pulsating flow, then relation (4) can be expressed in the form:

$$U^2 = A + B\,(\varphi_m)^N\,, \tag{6}$$

known as the King‘s equation [3, 4].

In practice, during measurements, the inverse characteristic representing the relation $\varphi_m = f(U)$ is more useful. In this case the inversed King’s equation is difficult to handle, because the determination of coefficients A, B, N requires an application of time consuming iterative procedures. Another problem is, that for correct approximation in the range of higher velocities ($v > 25$ m/s) the coefficient N should be expressed as a function of flow velocity $N = N(v)$ [6] what makes the problem still complicated. A convenient form of the inverse characteristic described by the 4th order polynomial was proposed by the TSI company [7]:

$$\varphi_m = E_0 + E_1U + E_2U^2 + E_3U^3 + E_4U^4, \tag{7}$$

where: $E_0,...,E_4$ – coefficients to determine experimentally.

In Figure 1, two exemple calibration curves, obtained for the employed anemometers, are presented. Their characteristics were described by Eq. (7).

Figure 1a shows that the fourth-order polynomial describes well the shape of the hot-wire characteristic. However, in some cases its application can make the approximation ambiguous. This particular case is shown in figure 1b. In the range of low specific mass flow rates, the approximating curve (1) is ambiguous (a characteristic “saddle” appears) and it cannot be used in this form. Usually this situation is met when the range of calibration is not sufficiently wide (compare ranges of the voltage U_{CTA} and the specific mass flow rate φ_m for cases 1a and 1b) and the number of points

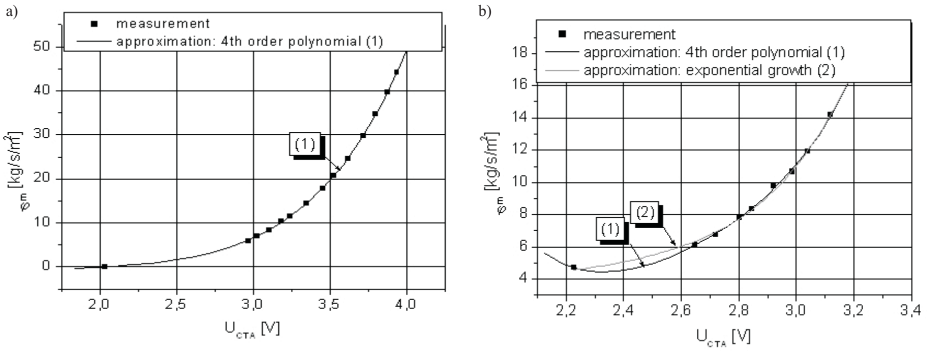

Fig. 1. Example calibration characteristics obtained for the employed hot-wire anemometers.

of calibration in the initial range of the characteristic is insufficient for its correct description.

This "empty" field of the characteristic (devoid of points from calibration) is often met because of difficulty in the generation of low flow rates.

If the approximation by means of the 4th order polynomial is ambiguous, an exponential growth function can be applied [8].

$$\varphi_m = A_0 + A_1 e^{\left(\frac{U}{B}\right)}. \tag{8}$$

This form of the characteristic (curve (2) – Fig. 1b) is free of this defect and yields correct approximation in the range of low mass flow rates, while its further plot is in fact equivalent to the 4th order polynomial (compare curves (1) and (2) – Fig. 1b). However, for high mass flow rates, the exponential curve becomes steeper than the polynomial, which makes the mass flow rate overestimated. Generally, the condition of similar mass flow rate ranges during calibration and measurement[5] should be satisfied to avoid these ambiguities.

2.2. Calibration of the CTA anemometer

The calibration was conducted on the same test rig as further measurements (see Fig. 2). Compressed air was used as a working medium. The principle of calibration is based on the assignment of the mass flow rate $\dot{m}$ (or the specific mass flow rate φ_m) to the voltage signal U at the anemometer output. This value is recorded by means of a standard flowmeter. The calibration process is performed under the conditions of the steady flow.

As the standard flowmeter, differential pressure probes (Annubar sensors) [9] were applied. They operate by sensing an impact pressure and a reference pressure through multiple sensing ports located on upstream and downstream sides of the probe. The resultant pressure difference is related to the flow velocity. The special diamond-like shape of the Annubar sensor establishes a fixed separation point in a wide range of Reynolds numbers. During additional measurements of static pressure and temperature upstream of the probe, it is possible to determine the mass flow rate $\hat{m}_{ref}$ averaged over the surface of the pipe cross section[6]. The analysis of measurement uncertainties for the mass flow rate shows that the Annubar accuracy does not exceed 1% of the measured value[7], which fully conforms to the producer declarations.

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[5] It should be noticed that the calibration is performed under the conditions of steady flow, while the measurement occurs under unsteady (pulsating) flow conditions. As a result, transient values of the mass flow rate may be distinctly higher than its mean value. During the tests, pulse amplitudes comparable to the mean value were often met.

[6] For the circulur cross-section of the straight pipe, the flow can be considered as axisymmetric.

[7] This uncertainty was calculated taking into account uncertainties of upstream pressure and temperature, differential pressure and Annubar flow coefficient.

During the calibration (as well as during the measurements), both hot-wire anemometers were mounted near the pipe axis to avoid an influence of the phenomena related to the presence of the boundary layer.

The calibration was carried out in the range of mass flow rates, which was supposed to be achieved during the measurements. Practically, it was limited by the maximal capacity of the volumetric compressor supplying the system with compressed air.

The temperature of the flowing gas was kept unchanged during the calibration (in the presented tests, it was stabilized at the level of approximately 40°C, which corresponded to the mean value expected for further measurements under the conditions of a pulsating flow).

Maintaining the temperature at a fixed level permits us to neglect the temperature influence on the hot-wire output signal (see Eq. 4).

The temperature was controlled by means of a set of electric heaters mounted in the air supply system.

In practice, the calibration was performed with a descending mass flow rate. The last point of the calibration (located extremely left on the characteristics – see Fig. 1) corresponded to the “no-flow” condition, which was accomplished by the complete closure of the throttle valve. The time of this measurement was minimized to avoid damage to the wire due to overheating.

High sensitivity of the hot wire probe is its specific feature. It is manifested by a very quick growth of the hot wire output signal under the influence of the mass flow rate increase. It can be observed on the inverse static characteristic (Fig. 1).

Although, generally, high sensitivity is considered as an advantage, in this case it causes a problem in precise determination of the characteristic plot in its initial range, since it is difficult to generate very low flows occurring in this field.

Consequently, between the first and the next point of the calibration, we obtain a large empty field (without points of calibration) worsening the accuracy in determination of the characteristics.

3. TEST RIG CONFIGURATION

A few different configurations of the test rig were used during the tests. Their common elements were: a system supplying compressed air, a heating system employing a set of electrical heaters, a pulse generator and pipes under test. Two different pipe lengths were tested: a “short pipe” (L_S = 0.544 m) and a “long pipe” (L_L = 1.246 m) (see Fig. 2). The distance between sections (0) and (3) was assumed as the pipe length. Both pipes had a circular section with a diameter of 42 mm.

In Figure 2a, a general arrangement of the test rig with a large volume tank mounted at the pipe outlet is presented. Figure 2b presents basic geometrical parameters of the essential part of the installation.

The generator described in [10] was applied as the source of pulsations. It operates like a rotating valve. Its rotor is driven by an electric motor controlled by a frequency inverter. The presented device allows for generating pulsations up to a frequency of 200 Hz.

Besides the system with a large volume tank, other arrangements were tested, namely: systems with a pipe either open at its end (exhaust directly to the atmosphere) or closed (in section 3) by a nozzle or a turbine of the automotive turbocharger.

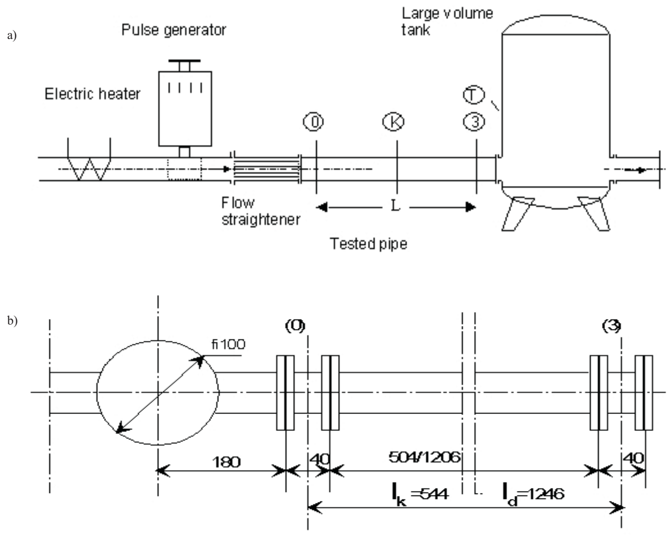

Fig. 2. General view of the test rig. A variant with a large volume tank (a) and an enlarged detail presenting the tested length of the pipe with its basic dimensions (b) are presented.

Transient pressure, temperature and specific mass flow rate were measured in control sections (0) and (3). An additional pressure measurement in the pipe mid-length section (K) was made as well[8].

Annubar sensor was mounted upstream the pulse generator and electric heater in order to avoid the backward influence of pulsations[9] .

4. TEST RESULTS

The results of specific mass flow rate measurements by means of the applied hot-wire probes are described below.

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[8] In the present paper only the specific mass flow rate measurements are described.

[9] Although the Annubar was generally used in steady flow conditions, it was also utilized during measurements in a pulsating flow as a control device.

Figure 3 presents diagrams of the specific mass flow rate φ_m in sections (0) and (3), recorded at the pulse frequency of 30 Hz. The acquired signals were approximated by means of the Fourier series as follows:

$$\varphi_m(t) = A_0 + C_1 \sin(2\pi f t + \phi_1) + ... + C_i \sin(i2\pi f t + \phi_i) + ... + C_n \sin(n2\pi f t + \phi_n), \quad (9)$$

where: A_0 – constant term (mean value of the signal); C_i – amplitude of the i^{th} harmonic; f – frequency of the first harmonic; ϕ_i – phase shift of the i^{th} harmonic; n – number of harmonics taken into account.

Signals were approximated by means of three first harmonics (Fig. 3b). Higher harmonics were considered as noise (their amplitudes are less than 5% of the mean value of the signal).

The analysis of the presented signals shows good coincidence of constant terms in the inlet and outlet sections (in the case under consideration, the difference is approximately 4%). It shows clearly that the calibration of both anemometers was correct. It indicates also that a possibility of determination of transient mass flow rate variations exists and it can be done on the basis of the measurements in one single point (without surveying the velocity field in the investigated section). In fact, the recorded signals should be representative for the whole control section. The balance of mean mass flow rates at the pipe inlet and outlet is the basic criterion of correctness of the applied procedures of calibration and measurement.

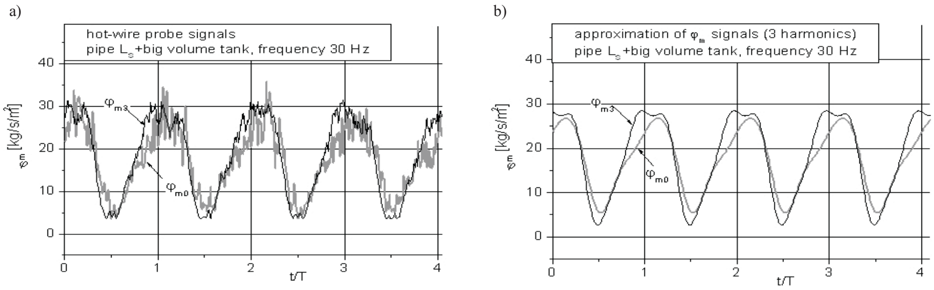

Fig. 3. Hot-wire probe signals φ_{m0} and φ_{m3} recorded in both the control sections (a) and their approximation by means of a Fourier series (b). Measurements for the short pipe (L_S) with a tank. Pulse frequency 30 Hz.

For the experimental verification of the assumptions made, measurements of the specific mass flow rate distribution in both inlet and outlet sections were made for a steady (corresponding to the calibration conditions) and unsteady flow. These distributions were obtained by surveying the flow field, displacing the probe radially from the pipe axis towards the pipe wall with a step of 2 mm (1 mm in the direct wall proximity).

Some selected results are presented in Figs. 4 and 5.

The signals $\varphi_m(l)$ recorded at successive radii (Fig. 4) show that variations of the specific mass flow rate are independent of the probe position in a relatively wide area around the pipe axis (Figs. 4a, b, plots from $r = 0$ to $r = 16$).

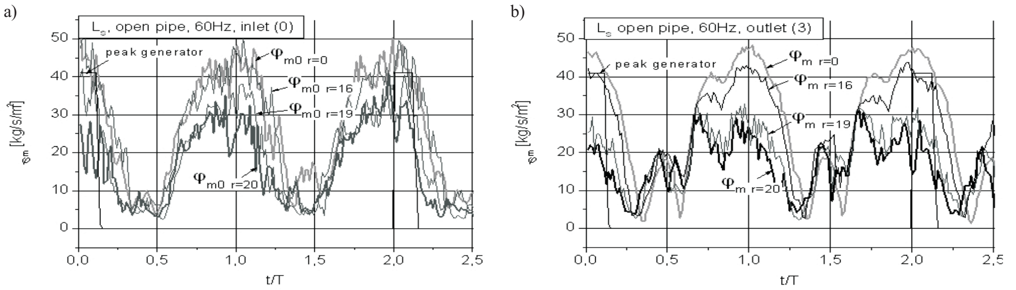

Fig. 4. Signals of the specific mass flow rate $\varphi_m(t)$ in inlet (a) and outlet (b) sections of the pipe for different probe positions. Measurements for the short pipe (L_S) open at the end. Pulse frequency 60 Hz. The peak generator signal corresponding to its full opening is presented as well.

This area – by analogy to the steady flow – can be referred to as the flow core. It can be observed in Fig. 5a, where a relation of the constant term A_0 (see Eq. 9) versus the radial probe position is presented.

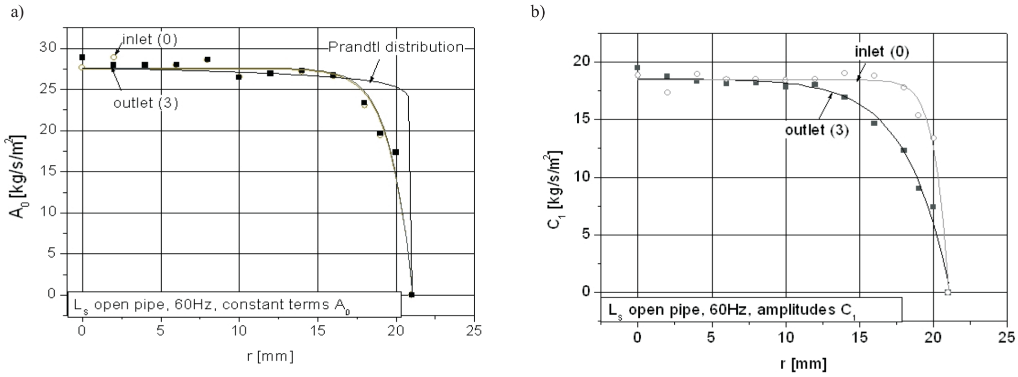

Fig. 5. Distribution of the mean specific mass flow rate A_0 (a) and the amplitude of the first harmonic C_1 (b) for the pulse frequency of 60 Hz.

Within the flow core, mean values of the specific mass flow rate correspond well to the Prandtl distribution [11], usually employed for the description of the flow fields in the circular section ducts:

$$A_0\,(r) = A_{0(r=0)}\left(1 - \frac{r}{R}\right)^{\frac{1}{n}}, \tag{10}$$

where: $A_{(0r=0)}$ – constant term in the pipe axis (r = 0); r – radial coordinate (distance from the pipe axis); R – pipe radius; $1/n$ – exponent determined experimentally (depending on the Reynolds number and pipe wall roughness).

Beyond the core, the Prandtl distribution is no more adequate (see Fig. 2a), so the approximation of the profile $A_0(r)$ by means of the Boltzmann equation was proposed:

$$A_0\,(r) = A_{0(r=0)} + \frac{\varphi_I - A_{0(r=0)}}{1 + e^{(r-r_I)/r_{II}}}, \tag{11}$$

where: φ_I,r_I,r_{II} – equation coefficients.

The shape of profiles $A_0(r)$ obtained for the inlet and outlet sections is very similar (see Fig. 2a). An insignificant dispersion of measuring points results from slight pressure fluctuations in the system of pipes supplying the test rig. The conservation of the specific mass flow rate profile along the pipe axis is extremely important in case of measurements executed in one single point of the pipe cross section (along the pipe axis in this case). It allows us to balance mean flow rates in both control sections, which has been mentioned at the beginning of Section 4.

Beyond the flow core, a rapid decrease in either the amplitude (Fig. 5b) or the mean value of flow velocity (Fig. 5a) takes place, nevertheless the character of successive plots as well as their phase shifts are maintained (Figs. 4a, b).

The state described for the pulse frequency of 60 Hz takes place in a wide range of tested frequencies, however for some frequencies of excitation the recorded signals of the specific mass flow rate look surprising. Figure 6a shows signals recorded for the short pipe in the system with a large volume tank at the frequency of 130 Hz. A huge disproportion of amplitudes at the pipe inlet and outlet is especially worth noting, as well as a quite different level of constant terms for these two sections. Also, the shape of the $\varphi)m3$ signal is surprising as additional local maxima appear. What is important, they have no analogs, neither in the recorded signals of pressure nor temperature. It is also worth noting that the transition between phases corresponding to main and local maxima (near zero) takes place without satisfying the condition of continuity of the first derivative.

The presented features of the recorded signal φ_{m3} indicate that in fact, in some parts of the pulsation period, an inversion of the flow direction takes place. This is the phase of the reverse flow characterized by negative velocities (see. Fig. 6b). As the CTA output signal depends on the quantity of heat exchanged between the wire and the flowing gas, the probe does not recognize the flow direction. An increase in the flow velocity is manifested by a more intensive heat exchange at the wire surface, which makes the CTA output signal higher, but a distinction of the flow direction is impossible.

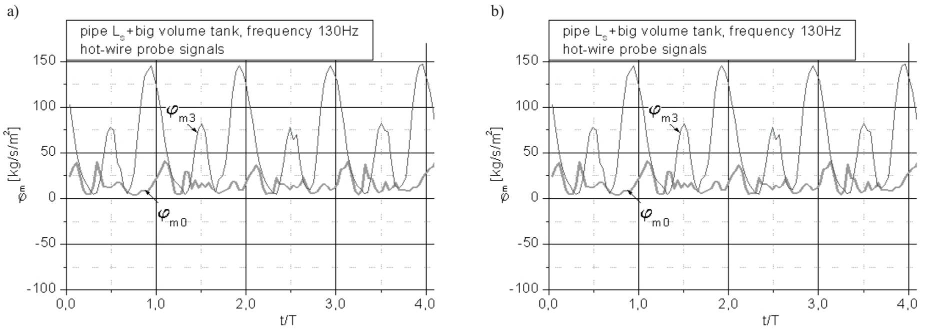

Fig. 6. Signals φ_m recorded by hot wire probes at the pulse frequency of 130 Hz (a) and the corresponding signals after the correction (b) considering the reverse flow of gas. Results for the short pipe (L_S) with a large volume tank.

In this sense, the CTA probe works as a full-wave rectifier converting a negative portion of the velocity signal into the positive one (Fig. 6a).

To reconstruct the real signal, the procedure of re-conversion of the rectified signal should be applied in respective parts of the pulsation period.

The effect of such a procedure is presented in Fig. 6b. As a result, the constant terms of velocity signals for inlet and outlet sections considerably approach one another.

The reverse flow phenomenon was confirmed by calculations performed by means of the model "$x - t$" describing the one-dimensional unsteady flow of gas and presented in detail in, e.g. [8, 12].

5. CONCLUSIONS

The presented method of measurements of the transient mass flow rate in ducts supplied with the pulsating flow allows us to receive quite precise information concerning the character of these variations, including the case of a reverse flow, which makes an interpretation of the recorded signals difficult.

The measurement of the specific mass flow rate conducted by means of hot wire probes in one single point of the pipe cross-section can be considered as representative for the whole section provided the velocity profile is uniform. This condition is nearly satisfied in case of the pulsating flow, as its character is distinctly turbulent, with an exception of the narrow region of the boundary layer in the direct proximity of the pipe wall. Under these conditions, the velocity profiles obtained for both inlet and outlet sections are very similar, which ensures balancing of the mean mass flow rate in these sections.

On the contrary, profiles of the specific mass flow rate obtained during the calibration, under the conditions of a steady flow cannot be considered strictly as uniform

– the obtained profiles are rather similar to those met for laminar flows, despite quite high Reynolds numbers (Re > 1×10^5).

In case of the reverse flow occurring at the pipe outlet for some excitation frequencies, a correction procedure of transient specific mass flow rate signals, which consists in taking into account the effect of an inversion of the velocity direction, should be applied.

Generally, the following conditions should be satisfied to assure correct transient mass flow rate measurement :

- similarity of mass flow rate ranges during calibration and measurement;
- balancing of the time-averaged mass flow rate representing the investigated section with the reference mass flow rate measured by a standard flowmeter[10];
- consideration of the effect of reverse flow occurring for resonance frequencies.

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[10] In the described configuration of pipes, the reference mass flow rate was measured in conditions of steady flow in the control section placed upstream of the pulse generator. If the flow is unsteady in the whole installation and the standard flowmeter works in conditions of unsteady flow, special procedures should be applied as described e.g in. [13, 14].