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LINEARITY ERROR AS COMPONENT OF A/D CONVERTER UNCERTAINTY

Analog-to-digital (A/D) converters are basic elements of measuring chains since they deliver measurement data being a carrier of digital information in measurement instruments and systems. Taking into account that the quality of the information depends on accuracy of data, one should evaluate the uncertainty of A/D conversion results. One of the uncertainty component is connected with the linearity error of an A/D converter. This error has to be contained in the uncertainty budget, however, it is possible if the error is described in probabilistic categories. The paper presents a way of obtaining such a kind of description, the basis of which is an analysis of A/D conversion as a quantization process consisting in a comparison of the measured quantity with a standard composed of quanta. The linearity error is treated as an effect of random distortion of quanta. A general model of a quantization result, containing both the quantization error and the error caused by quantum distortion, as well as the error generated by thermal noise, has been described. An analysis of correlation coefficients between these errors has been performed using the Monte Carlo method. A procedure of uncertainty calculation on the basis of the known error distribution has been presented.

Keywords: A/D converter, quantization, linearity error, correlation coefficient, error distribution, measurement result model, uncertainty

1. A/D CONVERSION AS A MEASUREMENT PROCESS

An A/D converter performs a measurement process on the principle of quantization, which means that a measured (quantized) quantity is compared directly with a multi-value standard being a sum of elementary standards, called quanta, the values of which are the same and considerably smaller than the measuring range of the A/D converter [1]. Such a multi-value standard is called a quantum structure standard and its way of constructing is different for every kind of A/D converter [2]. The standard used in the flash A/D converter, shown in Fig. 1, can be accepted as representative for a large number of A/D converter types.

The standard in the flash ADC is built up from the resistors R connected in series and supplied by the reference current source I_{ref} [3]. One quantum is equal to the voltage drop across the resistor R, therefore the voltage standard has as many values as the resistors exist. All the standard values are accessed at the same time, so every value of the measured voltage u can be compared with the standard at once. It causes that the time of the comparison process is limited only by the measured signal propagation in the ADC circuitry, which explains the name of the converter, i.e. flash. Such a comparison needs the number of comparators C equal to all values of the standard, which results in a quite sophisticated construction of the flash AD converter. For example, an 8 bit binary flash ADC contains the standard composed of $N = 2^8 = 256$ resistors delivering N standard voltage values changing from 1_q to N, where q is the quantum value and $q = RI_{\text{ref}}$. It means that the same number N of comparators is needed. A/D converter producers term a quantum value as 1 LSB (Least Significant Bit), so 1 LSB can be treated as the equivalent of the quantum q.

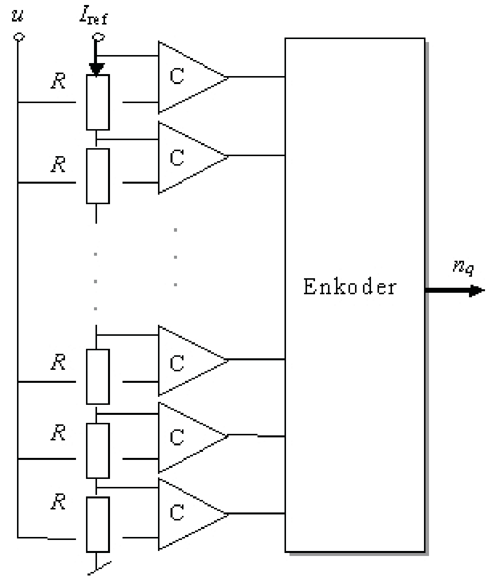

Fig. 1. Scheme of flash A/D converter.

The last element of the converter from Fig. 1, called an encoder, determines the number n_q of comparators that are in the state L. Such a state means that a value of the measured quantity u at one input of a comparator C is greater then the value of the partial standard, being the proper sum of quanta, at its second input. This way of an A/D converter functioning causes that n_q is the number of quanta, the sum of which is both less than the value of u and the nearest to it – such a kind of quantizer can be called as working according to the rule “the nearest less”. In this case the quantizer indication n_q may be written as:

$$n_q = \mathrm{ent}\left(\frac{u}{q}\right), \tag{1}$$

where **ent** denotes the function *entier*, which returns an integer value of its argument. The characteristic, described by Eq. (1), is shown in Fig. 2a for a 3-bit exemplary quantizer.

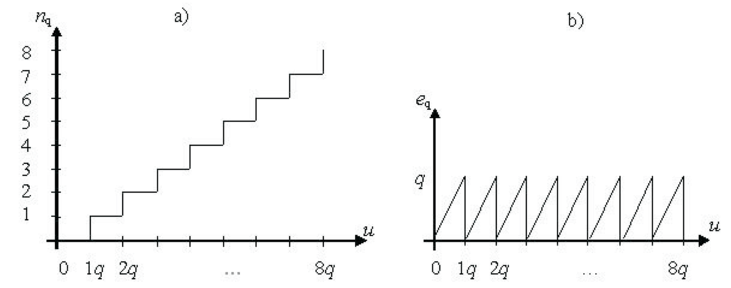

Fig. 2. Exemplary 3 bit quantizer, (a) characteristic, (b) quantization error.

Analysis of the quantization process leads to the conclusion [1] that the basic error caused by the standard with a quantum structure, called a quantization error, has the form

$$e_q = u - \breve{u}, \tag{2}$$

where

$$\breve{u} = qn_q, \tag{3}$$

is a designated indication, i.e. an indication expressed in a unit of the measured quantity u. The quantization error of the exemplary 3-bit converter is shown in Fig. 2b.

From the quantizer characteristic in Fig. 2a it results that the measured quantity u can change in the range from 0 to the value $u_r = Nq$. When one assumes that values of u are equally probable in every point of this range, the quantization error can be described in probabilistic categories. In this case the error distribution is rectangular in the range from 0 to q, as shown in Fig. 3a [1].

The probability density function $g(e_q)$, presented in Fig. 3a, is not symmetrical with respect to the vertical axis, which means that every quantization error contains a systematic component. This component is equal to the expected value of the function $g(e_q)$, so it can be calculated as:

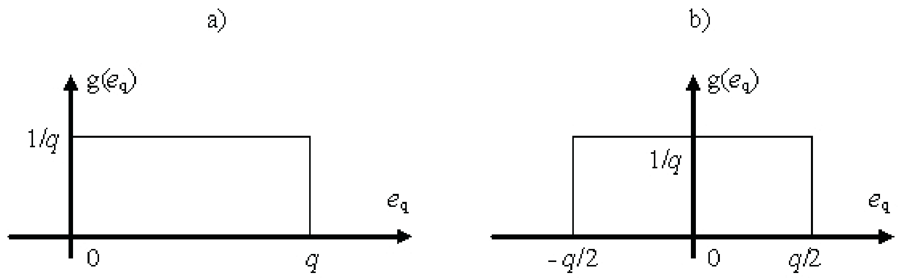

Fig. 3. Distributions of a quantization error, (a) for the quantizer with the characteristic shown in Fig. 2a, (b) for the quantizer after correction of the systematic component of a quantization error.

$$E\left(e_q\right) = \int_{-\infty}^{\infty} e_q g\left(e_q\right) de_q = \int_{0}^{q} e_q \frac{1}{q} de_q = \frac{q}{2}. \tag{4}$$

After subtraction of the obtained expected value $q/2$ from every value of the quantization error its probability density function takes the symmetrical form shown in Fig. 3b. When one takes into account Eq. (2), such an operation is equivalent to addition of $q/2$ to every quantization result, since:

$$e_{\mathrm{q}} - \frac{q}{2} = u - \left(\breve{u} + \frac{q}{2}\right) = u - \hat{u}. \tag{5}$$

The corrected quantization (measurement) result $\hat{u}$ is the best evaluation of the measured quantity value u possible to obtain if a quantizer is used as the measuring instrument. To the next consideration, it is assumed that every result is corrected in the way described by Eq. (5).

The characteristic of the quantizer, the results of which are corrected in the described way, can be given by the expression:

$$n_q = \mathrm{ent}\left(\frac{u}{q}\right) + 0,5, \tag{6}$$

which is used as the mathematical model of a quantizer in simulation experiments performed for the goals of the paper. This model describes a digitally corrected quantizer, the characteristic of which is obtained by adding 0,5 to every indication of the quantizer modeled by Eq. (1). Such a quantizer is equivalent to these ones corrected physically (by using the proper circuits [3]) if the quantization error distribution, shown in Fig. 3b, is taken into account.

2. LINEARITY ERROR OF THE QUANTIZER

The quantizer described by Eq. (6) can be treated as ideal in the sense that all quanta have the same values. In real quantizers the value of every quantum is different because of technological limitations in their production process. For example, the resistors of the flash ADC from Fig. 1 are usually cut on a semiconductor surface by a laser ray, which causes that their resistances are different dependently on local homogeneity of the semiconductor and precision of cutting. Such a dispersion of quantum values causes that the standard used in the quantization process is not accurate, which is a source of one more error in this process.

To analyze the rise of the error caused by quantum dispersion (shortly called a dispersion error), let us start with the assumption that the structure of the standard does not change. It means that the standard is composed from quanta, the values of which can be described as a series $q(1)$, $q(2)$, …., $q(N)$, where N is the total number of the quanta (such a standard is used in the flash A/D converter from Fig. 1). The random nature of the dispersion is a cause of differences between every quantum value and the nominal quantum value q_{nom}. The difference can be described as a random error:

$$\delta(n) = q(n) - q_{\mathrm{nom}}, \tag{7}$$

where n is the quantum number, $n = 1, 2, \ldots, N$, and $q_{\mathrm{nom}} = u_r/N$, u_r is the range of the quantizer.

As the standard of a quantizer is composed of quanta, the succeeding partial standard values are sums of quantum values. The successive sums are described by the recursive rule:

$$\begin{aligned} &U_{\mathrm{ref}}(1) = q(1), \\ &U_{\mathrm{ref}}(n) = U_{\mathrm{ref}}(n-1) + q(n) \;\; n = 2, \ldots, N. \end{aligned} \tag{8}$$

If all quanta have the same values q_{nom}, rule (8) takes the form:

$$U_{\mathrm{ref}}(n) = nq_{\mathrm{nom}}, n = 1, \ldots, N, \tag{9}$$

but when they are burdened by errors (7), their values can be written as:

$$\begin{aligned} &\tilde{U}_{\mathrm{ref}}(1) = q_{\mathrm{nom}} + \delta(1), \\ &\tilde{U}_{\mathrm{ref}}(n) = \tilde{U}_{\mathrm{ref}}(n-1) + q_{\mathrm{nom}} + \delta(n) \;\; n = 2, \ldots, N. \end{aligned} \tag{10}$$

Defining the partial standard error as:

$$\delta_{\mathrm{ref}}(n) = \tilde{U}_{\mathrm{ref}}(n) - U_{\mathrm{ref}}(n), \tag{11}$$

on the basis of Eqs. (7), (8) and (10) one obtains a description of the partial standard errors in the form:

$$\begin{aligned}
&\delta_{\text{ref}}(1) = \delta(1), \\
&\delta_{\text{ref}}(2) = \delta(1) + \delta(2), \\
&\vdots \\
&\delta_{\text{ref}}(N) = \delta(1) + \delta(2) + \ldots + \delta(N).
\end{aligned} \tag{12}$$

Eqs. (12) show that dispersion errors burdened the quanta with lower numbers cumulate in the partial standards with higher numbers. All errors are independent and they are of the same distribution, therefore, if one takes into account the central limit theorem [4], distributions of their sums for n ≥ 3 can be treated in practice as normal with the variance:

$$\sigma_{\text{ref}}^2 = n\sigma^2, \tag{13}$$

where σ^2 is the variance of the error δ. On should notice that in this case the shape of the dispersion error does not have to be known.

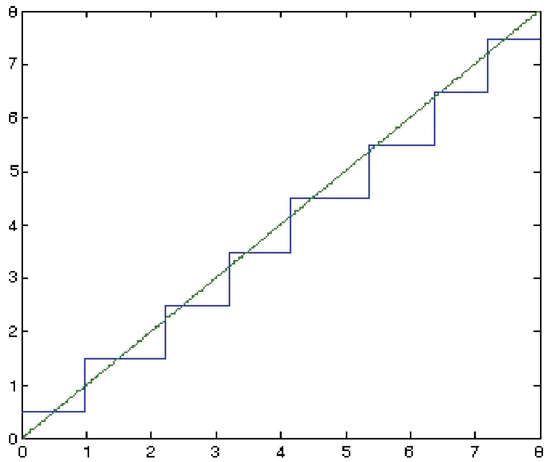

Fig. 4. Characteristic of exemplary 3 bit A/D converter with quanta burdened distortion errors.

Differences of the quantum values cause that the quantizer characteristic is not as regular as that shown in Fig. 2a. An exemplary characteristic of a 3-bit A/D converter with standard composed of quanta burdened errors with rectangular distribution in the range from $-\varepsilon$ to ε, where $\varepsilon = dq_{\text{nom}}$, d is the relative dispersion and d = 30%, is shown in Fig. 4. The nominal value of the quantum q_{nom} = 1V. Figure 5 presents the total error of the exemplary A/D converter as a function of the measured quantity u. The total error values are obtained by subtraction of the characteristic from Fig. 4 and the ideal one being the line connecting two points: 0 and $u_r = q_{nom}N$, also shown in this figure.

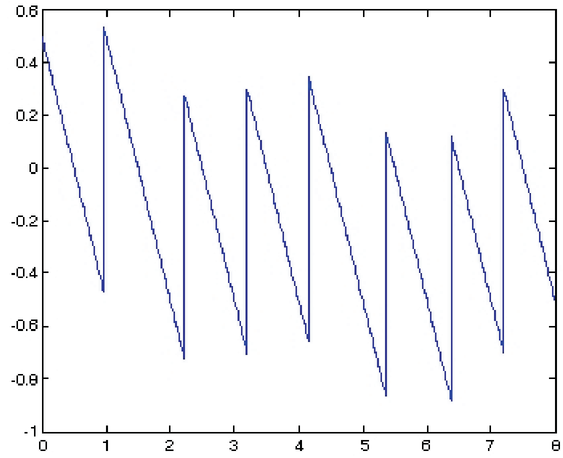

Fig. 5. Total error of exemplary 3 bit A/D converter.

The total error e is a composition of the quantization error and the standard error. Its maximum absolute value is called linearity [3], integral linearity error or nonlinearity error and expressed in application notes as one of the parameters of A/D converters, usually as a fraction of 1 LSB. Such a parameter can be used to compare different constructions of A/D converters (the better converter has a lower linearity error) but it is useless when one wants to determine the inaccuracy of the applied converter. A solution of this problem can be obtained on the basis of the model of a measurement result, which contains the partial error described in probabilistic categories. The next section is devoted to a presentation of this solution.

3. PROBABILISTIC MODEL OF THE QUANTIZATION RESULT

As shown in paper [1], a mathematical model of the measurement result can be deduced directly from an analysis of the quantization process. The model has the form:

$$u = \hat{u} + e, \tag{14}$$

which means that the single measurement result u is the sum of the evaluation $\hat{u}$ of the measured quantity value (devoid systematic components of errors) and a realization of the total error e. In general, the error e can be the sum of I random partial errors:

$$e = e_1 + e_2 + \ldots + e_I. \tag{15}$$

The distribution of every error is known and described by a probability density function, with the expected value equal to zero, or its equivalent in the form of a histogram. Possible relations between the errors are given by correlation coefficients which describe the dependence between variances of partial errors in determination of its resultant error.

The variance of a random error e is defined by the expected value:

$$\sigma^2 = E\left[e - E(e)\right]^2, \tag{16}$$

which, in the case when the error e is a sum of two partial errors e_1 and e_2 with expected values $E(e_1)$ and $E(e_2)$ equal to 0, takes the form:

$$\sigma^2 = E\left[e_1 - E(e_1) + e_2 - E(e_2)\right]^2 = E\left[e_1 + e_2\right]^2. \tag{17}$$

Transforming Eq. (17), one obtains:

$$\sigma^2 = E\left(e_1^2 + 2e_1e_2 + e_2^2\right) = E\left(e_1^2\right) + 2E(e_1e_2) + E\left(e_2^2\right) = \sigma_1^2 + 2\mathrm{cov}(e_1, e_2) + \sigma_2^2, \tag{18}$$

where σ_1^2, σ_2^2 are variances of the errors e_1 and e_2, respectively, and $\mathrm{cov}(e_1, e_2)$ is its covariance [5]. The normalized covariance

$$r_{1,2} = \frac{\mathrm{cov}(e_1, e_2)}{\sigma_1\sigma_2} \tag{19}$$

is called the correlation coefficient.

Calculation of the correlation coefficient of two errors is usually performed on a series of couples of realizations of these errors obtained as the result of an experiment. In the situation when such a series is not given one can determine the coefficient using the equation

$$r_{1,2} = \frac{\sigma^2 - \sigma_1^2 - \sigma_2^2}{2\sigma_1\sigma_2}, \tag{20}$$

obtained on the basis of Eqs. (18) and (19).

3.1. Standard error as a component of A/D converter total error

At the beginning let us consider such a situation when one measurement result is obtained by using a quantizer, the standard of which consists of quanta burdened errors caused by distortion. In this case the total error of the result is a composition of two partial errors: the quantization error e_q and the standard error e_d caused by differences of quantum values (the linearity error can be treated as a parameter of the total error). As the quantization error distribution is known (see Fig. 3b), determination of the distribution of the standard error and the correlation coefficient of these two errors is needed. It can be performed in a simulation way using the Monte Carlo method [6].

The standard error can be interpreted as additional to the quantization error arising when quanta with nominal values are used. It means that values of the distortion error can be calculated by subtraction of the quantization error from the total error. The values obtained in this way for the exemplary 3-bit quantizer are presented in Fig. 6. One can notice that absolute values of the standard error are integer products of a quantum value, i.e. are equal to 0, $1q$, $2q$,… dependently on the quantum distortion and the number of A/D converter bits.

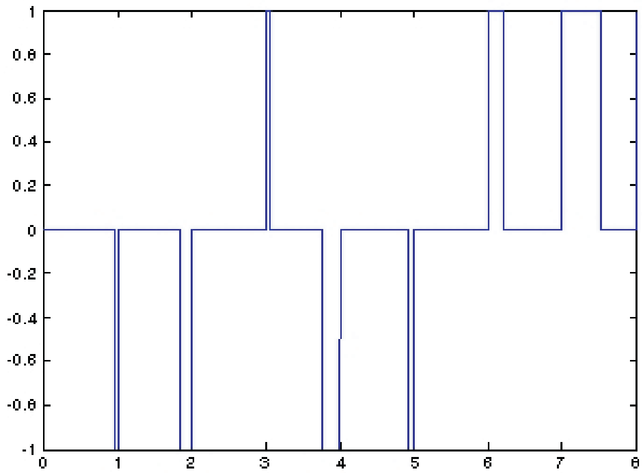

Fig. 6. Standard error of the exemplary 3 bit quantizer as function of measured quantity.

In order to carry out an analysis of the standard error properties, the n bit flash A/D converter, as in Fig. 1, is taken as the object of simulation experiments. Let us assume that its measurement range u_r = 1,024 V, which means that the nominal quantum value $q_{nom} = u_r/2^n$. To start with let us determine a histogram of the standard error for the 8-bit converter by using the experiment performed in 100 000 steps. At every step, at first, a value of the measured quantity u is taken from a population with a rectangular distribution in the measurement range. Next, this value is quantized by comparison with the standard, the quanta of which are burdened with errors caused by distortion described by the rectangular distribution in the range from $-\varepsilon$ to ε, where $\varepsilon = dq_{nom}$, d is the relative distorsion, and $d = 3\%$.The indication n_q is corrected by adding $q_{nom}/2$ and multiplied by q_{nom} that permits obtaining one quantization result $\hat{u}$. Having given this result, the total error value is calculated from the equation:

$$e(i) = u(i) - \hat{u}(i) = u(i) - q_{nom}\left[n_q(i) + 0,5\right] \tag{21}$$

and then, the standard error as:

$$e_s(i) = e(i) - e_q(i) = e(i) - q_{nom}\left[\mathrm{ent}\left(\frac{u(i)}{q_{nom}}\right) + 0,5\right], \tag{22}$$

where i is the step number, and $i = 1, \ldots, 100\,000$. The calculated set of the standard error values is shown in Fig. 7 as a histogram.

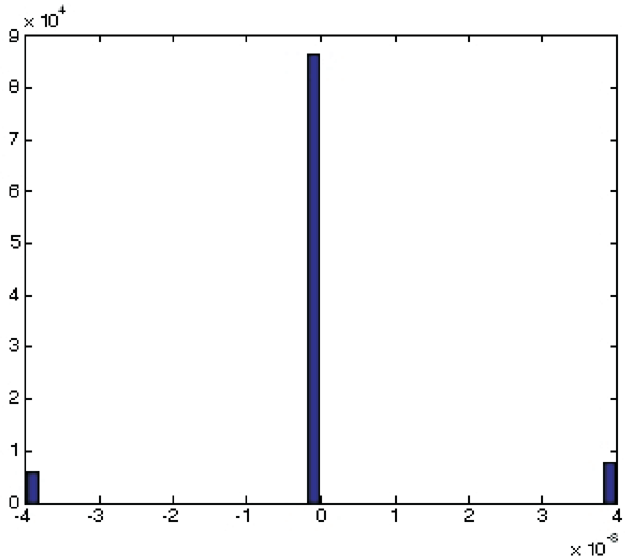

Fig. 7. Standard error histogram of exemplary 8 bit flash A/D converter.

Calculation of the correlation coefficient for errors of the exemplary 8-bit flash converter gives a result different from 0. It means that determination of the total error variance according to Eq. (18) requires, except for suitable variances, a value of this coefficient. The basic question one should answer is whether the correlation coefficient depends on the variances or other parameters of the quantization process.

Table 1. Correlation coefficient values r_{qs} of quantization and standard error in relation to number of A/D converter bit n, relative quantum distortion $d = 3\%$.

n	8	10	12
r_{qs}	– 0.49	– 0.47	– 0.32

To obtain such an answer, two experiments have been carried out for a group of 100 A/D converters. Every converter includes a different standard composed of quanta made by using the same technology, which means that they are burdened with random errors of the same properties. For every converter, 1000 couples, composed of the

Table 2. Correlation coefficient values r_{qs} in relation to values of relative quantum distortion d for 8 bit A/D converter.

d[%]	1	2	3	4	5	6	7
r_{qs}	– 0.37	– 0.45	– 0.49	– 0.51	– 0.49	– 0.45	– 0.43

quantization error and the standard error values, are calculated on the basis of Eqs. (21) and (22) for values of the measured quantity taken from the converter input range accordingly with the rectangular distribution. The first experiment has been aimed at checking the coefficient dependency on the number of A/D converter bits. The results are presented in Table 1 while Table 2 shows the correlation coefficient dependence of the 8-bit A/D converter on the relative distortion. An exemplary histogram of the total error composed of these two errors is shown in Fig. 8.

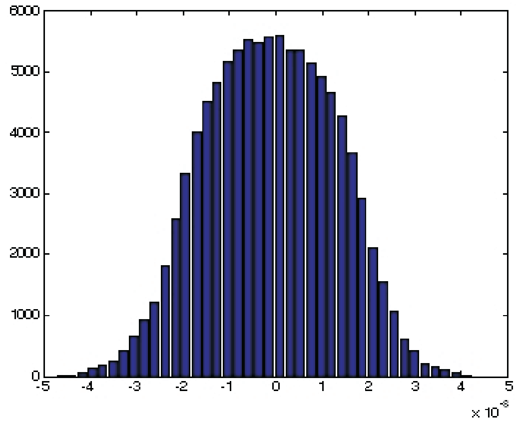

Fig. 8. Histogram of the total error of 8 bit A/D converter with relative quantum distortion d = 3%.

3.2. Influence of A/D converter adjustment on standard error

Every utilized A/D converter has to be adjusted periodically according to recommendations of its producer. The idea of the simplest way of adjustment consists in reducing to zero the errors in two points of the A/D converter characteristic: at the beginning and at the end of the converter range. These errors are compositions of several different errors [2], one of them is the standard error (for an unipolar A/D converter as in Fig. 1 it is the error of the last partial standard $U_{\text{ref}}(N)$). If one assumes that the standard error dominates, the adjustment consists in such a change ΔI_{ref} of the reference current that $U_{\text{ref}}(N)$ is equal to the voltage range u_r. It means that every quantum is corrected by adding a suitable value:

$$\Delta q\,(i) = \Delta I_{\text{ref}} R\,(i) = \frac{U_{\text{ref}}\,(N) - u_{\text{z}}}{\sum\limits_{i=1}^{N} R\,(i)} R\,(i)\,, i = 1, ..., N. \tag{23}$$

Table 3. Total standard deviations ratio: with (σ_{a}) and without (σ) adjustment in relation to bit number n of A/D converter.

n	8	10	12
σ_{a}/σ	0.88	0.75	0.62

Such a correction causes that errors of partial standards take smaller values. To investigate the influence of the adjustment on the standard error, a simulation experiment has been realized for a group of A/D converters with the same parameters as described in the previous section. The experiment results are presented in Table 3. Figure 9 shows a histogram of the total error in the case when the adjustment has been performed.

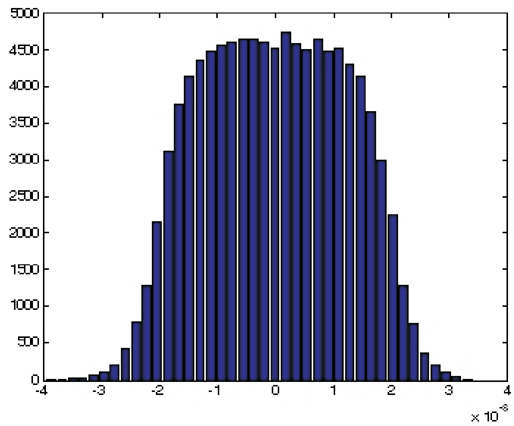

Fig. 9. Histogram of total error of exemplary 8 bit A/D converter after adjustment, relative quantum distortion d = 3%.

3.3. Influence of noise on A/D converter total error

Except of the described two kinds of error, there is a one more error that can considerably influence the inaccuracy of a quantization process. This error is caused by thermal noise generated mainly by resistors in the A/D converter circuitry. As it has been calculated by using a simulation experiment realized in the same way as described in section 3.1, the correlation coefficient of the noise error and the quantization error is close to zero. Therefore, the noise error can be treated as additive to the other errors in the model (14) of the quatization result. An exemplary histogram of the A/D converter total error composed of the investigated three kinds of error is shown in Fig. 10. From the histogram it results that even noise with relatively low standard deviation (0,1q) causes "smoothing" of the total error distribution to a shape which can be treated in practice as Gaussian (normal).

4. UNCERTAINTY OF A SINGLE QUANTIZATION RESULT

A probabilistic model of a measurement result allows determination of a quantizer inaccuracy as uncertainty according to recommendations of the guide [7], which defines it as a parameter that characterizes a dispersion of values which can be attributed to a measured quantity. If one takes into account the measurement result model (14), this definition can be written in the mathematical form

$$\mathbf{Pr}\left[|u - \hat{u}| \leq U\right] = \alpha, \tag{24}$$

which determines the uncertainty U as such a parameter that probability **Pr**[ ] equals the confidence level α (usually one takes $\alpha = 0.95$ [7]). From Eqs. (14) and (24) it results that having given the probability density function $g(e)$ of the error e, one can calculate the uncertainty U on the basis of the functional

$$\int_{-U}^{U} g(e)de = \alpha, \tag{25}$$

with the assumption that $g(e)$ is symmetrical in relation to the vertical axis.

After determination of the uncertainty, the measurement result can be written as an interval. From Eq. (24) it results that the inequality

$$|u - \hat{u}| \leq U, \tag{26}$$

is satisfied with probability α. Expression (26) can be written as

$$\hat{u} - U \leq u \leq \hat{u} + U, \tag{27}$$

or in the interval form

$$u = [\hat{u} - U, \hat{u} + U]. \tag{28}$$

To calculate the uncertainty of a measurement result, the distribution of its total error should be determined. For example, if the distribution is given by the histogram as in Fig. 10, the uncertainty calculated by using formula (25) has the value

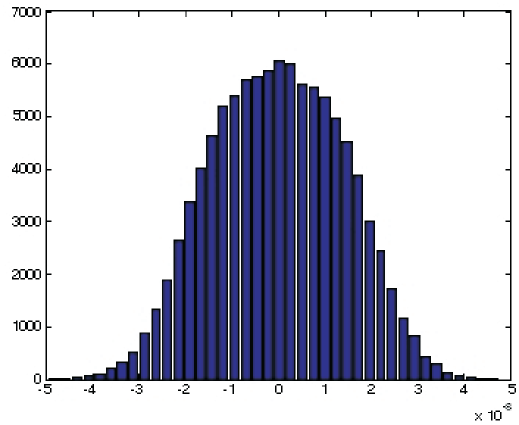

Fig. 10. Histogram of total error of exemplary 8 bit A/D converter, relative distortion $r = 3\%$, standard deviation of the noise error with normal distribution $\sigma = 0,1q_{\text{nom}}$.

$U = 12.10^{-3}$ V. Let us assume that the indication of an A/D conversion is $n_{\text{q}} = 204$, which means that the quantization result equals $u = 204.0{,}004\text{V} = 0{,}816\text{V}$. Correction of the systemic component of the quantization result gives the value of the result $\hat{u} = u + 0,5q = 0,816 + 0,5.0,004\text{V} = 0{,}818$ V. Taking into account the uncertainty value, this result can written in the interval form (28) as:

$$u = [0,818 - 0,012, 0,818 + 0,012]\,\text{V} = [0,806,\ 0,830]\,\text{V}. \tag{29}$$

The model (14) of the measurement result can be applied in the situation when the quantized signal is burdened with errors connected with elements that precede an A/D converter in the measurement chain, such as a sensor, amplifiers, sample/hold circuit etc. The analysis results presented in paper [1] show that there is no correlation between the described errors of a quantizer and the errors of the quantized signal. If there is a need to check the correlation between selected errors one can carry out such a simulation experiment as performed in the paper. The total error of the measurement result can be determined in this case by using convolution of partial error distributions. It is possible to calculate the resultant uncertainty directly on the basis of partial uncertainties using the mathematical apparatus described in [8].

5. CONCLUDING REMARKS

Investigation results presented in the paper let us to draw the conclusion that the error caused by distortion of quantum values is of random nature. Its influence on an A/D converter characteristic is traditionally described by a parameter called the linearity error. Knowing such parameters of different A/D converters one can evaluate the characteristic which is more linear but the concrete value of the linearity error cannot be used in the uncertainty calculation of an A/D conversion result It is possible when the distortion error is described in probabilistic categories as a component of the quantization result model. Because of nonlinearity of the quantization process this error is correlated with the quantization error, which causes that the composition of these two errors is a quite sophisticated process and its use in practice is difficult. A simpler solution consists in the determination of the total error distribution using the Monte Carlo method in realizing simulative experiments (if parameters of the quantum distortion are given) or measurement experiments [9]. This way has another advantage since it enables to take into account errors caused by thermal noise generated in the A/D converter circuitry. In conclusion, one can say that the total error distribution obtained in this way is the best description, from a practical point of view, of metrological properties of an A/D converter because it permits a relatively simple calculation of the uncertainty of the A/D conversion result. I think that the total error distribution should be included in application notes of every A/D converter.

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