METROLOGY AND MEASUREMENT SYSTEMS
VOL. XV, NUMBER 4 (2008)

ANNA DOMAŃSKA

Poznan University of Technology
Faculty of Electronics and Telecommunications
Poland, e-mail: domanska@et.put.poznan.pl

ELIMINATION OF NONLINEARITY WITH DISCONTINUITIES (OF THE DEAD ZONE) FROM STATIC CHARACTERISTIC OF CONVERTER

The characteristics of real converters, including sensors, are – to a smaller or larger extent – nonlinear. Nonlinearity may result from the technologies applied to produce the converter or from the type of processing to be realized by the converter. With respect to the mathematical model describing the characteristics, they can be characteristics with smooth nonlinearity or nonlinearity with discontinuities. The author presented the problem of elimination of nonlinearity with discontinuities from the converter characteristic, which appears in the small-value area, i.e. in the dead zone, with a method utilizing the technique of added noise (dither). The properties of a converter with modified characteristic are shown: the ability to process input signals that previously could not be processed because their values were within the dead zone; removal of discontinuities in the output signal within the range of small values.

Keywords: input-output characteristic, dead zone, nonlinear distortion, dither.

1. INTRODUCTION

There is a desired feature of converters, including sensors: linearity of the input-output characteristic. The characteristics of real converters, including sensors, are – to a smaller or larger extent – nonlinear. If nonlinearity has an unacceptable influence on measurement accuracy, characteristics are linearized. If nonlinearity appears, it means that sensitivity in the measurement range is variable. Measurement accuracy is then also variable, depending on the value measured [1].

Nonlinearity may appear in the whole range of input values (large scale nonlinearity), or only in certain sub-ranges for small, medium or large values (small scale nonlinearity). With respect to a mathematical model describing the characteristics, characteristics with nonlinearities can be divided in two types: characteristics with smooth nonlinearity (Fig. 1a, Fig. 1b) or nonlinearity with discontinuities (Fig. 1c, Fig. 1d, Fig. 1e.)

[1] Received: October 11, 2008. Revised: November 20, 2008.

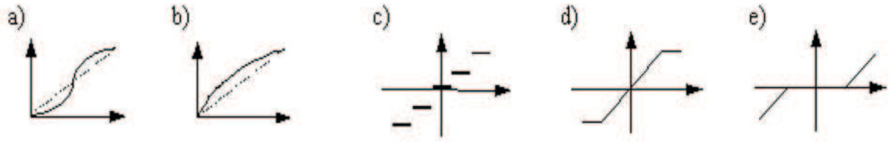


Fig. 1. Types of nonlinearity in the input-output characteristics of converters: smooth – a,b, with discontinuities – c,d,e.

Nonlinearities can be reduced with the use of system or program methods. The factors determining the selection of the method are first of all the technology applied to produce the converter and cost-performance analysis.

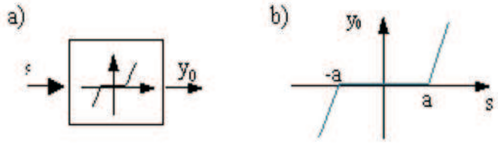
The paper presents the problem of elimination of nonlinearity with discontinuities from the converter characteristic, which appears in the small value area, i.e. in the dead zone, Fig. 1e. In general, this is not an easy task because of the type of nonlinearity and the local character of linearization. A method utilizing the technique of added noise (dither) was applied to this purpose. The development of this technique started at the beginning of the 60-ties. First applications concerned quantizing converters [2,3,4]. Up to now, the applications concerning quantization and AD converters have been most developed (hundreds of publications) [5,6,7,8]. Apart from A-D conversion, the dither linearization method found application: in nonlinear control systems with feedback (e.g. stability improvement) [9,10,11], in switching systems (e.g. sliding-mode control of power converters) [12], in converters of MEMS type (e.g. to improve the SNR, especially at small accelerations) [13].

The linearization problem presented in the paper concerns converters operating in systems without feedback.

Chapter 2 discusses the properties of a converter with a dead zone. Chapter 3 describes the rule of elimination of the dead zone from converter characteristic with the use of the dither method, and the rule of selection of the dither signal and the selection of the system eliminating any random component from the processing result (a necessary element connected with the realization of the method). Chapter 4 shows the properties of the converter after the characteristic was modified. Its new potential was presented: processing of input signals that previously could not be processed because their values were within the dead zone; removal of nonlinearities in the output signal, appearing in the range of small values. Chapter 5 gives an analysis of the influence of the applied elimination method on converter accuracy. Chapter 6 concludes the considerations.

2. PROPERTIES OF CONVERTER WITH A DEAD ZONE CHARACTERISTIC

It is impossible for a converter with a dead zone characteristic to process those values of signals that do not exceed the zone range. The denotation and characteristic of such a converter are shown in Fig. 2.



c)

$$y_0 = \begin{cases} k \cdot (s - a), & s > a \\ 0, & |s| \le a \\ k \cdot (s + a), & s < -a \end{cases}$$

Fig. 2. Converter with characteristic with dead zone: a) symbol, b) input-output characteristic, c) processing equation.

Consequently, in the output signal of the converter nonlinearities appear in the small- value area, or the output signal does not appear at all if values of the input signal are smaller than the range of the dead zone. Both situations are illustrated in Fig. 3. In the considered case $k = 1$, $a = 0.2$.

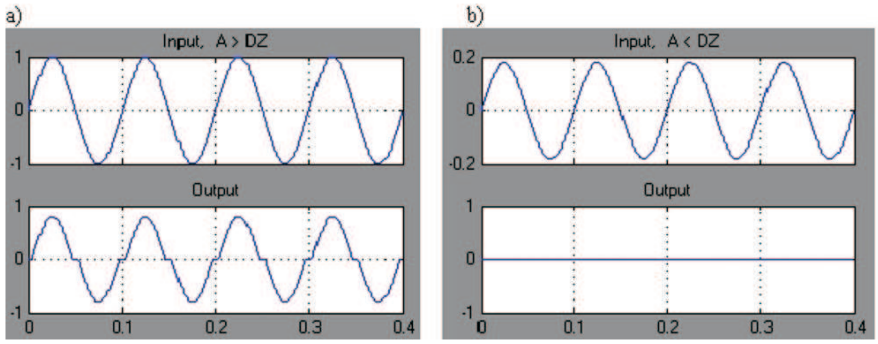


Fig. 3. Input and output signal of the converter, $-0.2 \le \mathrm{DZ} \le 0.2$, a) A= 1, b) A= 0.18.

The appearance of nonlinearity in the output signal is identical with a change (“enhancement”) of its spectrum – as compared with the spectrum of the prime signal, i.e. with its nonlinear distortion. The spectra of both signals – input and output from Fig. 3a – are shown in Fig. 4a and 4b.

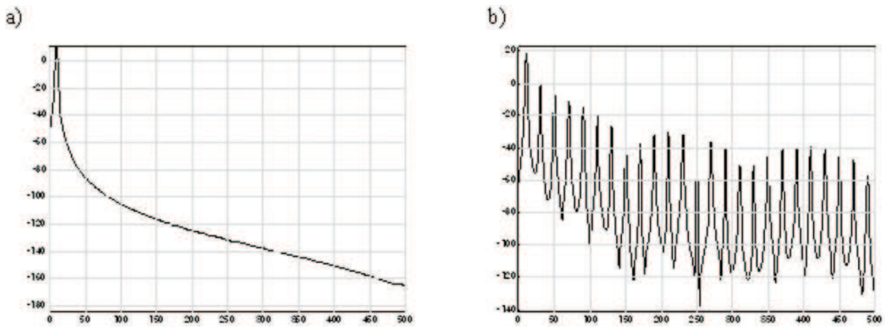


Fig. 4. Converter with dead zone: a) spectrum of input signal, b) spectrum of output signal.

On the other hand, the fact that it is impossible to process signals with small values is a limitation that enforces us to apply solutions usually causing considerable extension of the measurement channel.

3. RULE OF ELIMINATING THE DEAD ZONE FROM CONVERTER CHARACTERISTIC WITH THE USE OF THE DITHER METHOD

In order to eliminate the dead zone from a characteristic by using the dither method, we add another signal (dither) to the input signal of the converter, we process them together, and then average / filter the result. The idea of such linearization is shown in the diagram in Fig. 5.

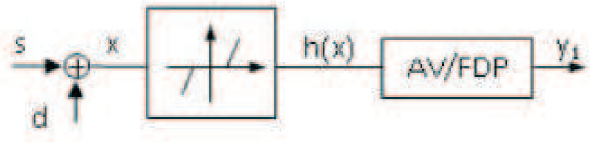


Fig. 5. Linearization of converter characteristic with a dither signal.

3.1. Selection of the dither signal

The objective of the additional dither signal is "taking out" of values of the prime input signal, contained in the dead zone, beyond the zone. Theoretically, every noise with a suitable variance or a periodic signal (sinus, triangle, rectangle), having a suitable range of values and suitable frequency (considerably larger than the main signal frequency), selected according to the size of dead zone, can play the role of dither [9,10, 11]. Practically, however, nearly always it is noise with rectangular or gaussian probability density function (PDF), for the following reasons:

– with respect to the extremely complex structure, in the envelope of the signal altered with such dither the form of the prime signal is best preserved,
– with respect to many possibilities of further elimination of noise (averaging, filtration, self-contained filtration).

The considerations presented here relate to the case of applying dither being white noise with zero average value and with rectangular and gaussian PDF. If the converter has a symmetric characteristic with a range of the dead zone $-a \leq \mathrm{DZ} \leq a$, the range of dither changes should be at least $2a(D_{pp} \geq 2 \cdot a)$.

When dither (d) with span D is added to the prime input signal (s) the values of which are contained inside the dead zone, the values of the converter input signal ($s \pm d$) begin to stick out of this zone to a degree depending on the position of the values of the prime input signal inside the zone. Therefore, a signal appears at the converter output. The signal consists of a random component coming from dither and a slowly-variable component from the prime input signal. As a result of averaging/filtration of the output signal, the random component is eliminated, and the result obtained is equivalent to processing of the prime input signal in a converter with characteristic y_1 from Fig. 7.

3.2. Random component elimination system

The purpose of the AV/LPF system is to eliminate the random component from the converter output signal. It can be a system realizing the averaging (AV), or a typical low-pass filter (LPF). If dither was a narrow-band signal with the range located outside the converter band, then the elimination takes place as a result of self-contained filtration, and the role of the elimination system is played by the converter itself.

The elimination can be realized with an analog or digital method, depending on the form or further purpose of the signal outgoing from converter $h(x)$, Fig. 5.

In the present considerations an analog Bessel low-pass filter was used. It is a type of filter in which best linearity of the phase characteristic is obtained, compared to other types of filters of the same order.

4. MODIFIED CONVERTER CHARACTERISTIC

The application of the method described removes the nonlinearities in the output signal; it also makes it possible to process input signals the values of which do not exceed the dead zone. Both cases are illustrated in Fig. 6.

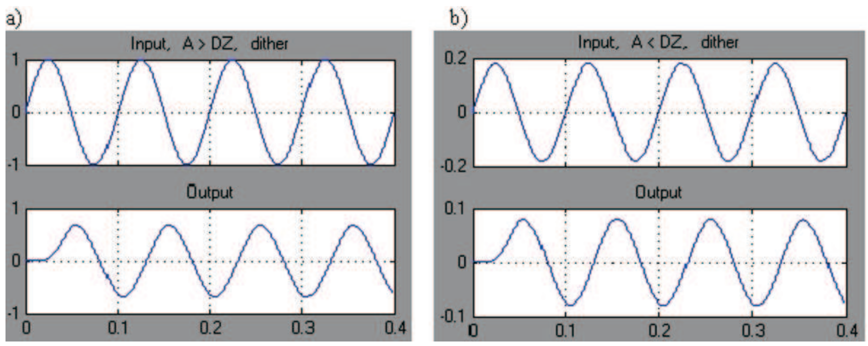


Fig. 6. Input and output signal of converter with dither D_{pp} = 0.4, -0.2 ≤ DZ ≤ 0.2, a) A= 1, b) A= 0.18.

Differences between processing in systems from Fig. 2a and from Fig.5 can be noticed when comparing corresponding pairs of signals: Fig. 3a and Fig. 6a as well as Fig. 3b and Fig. 6b. In the system from Fig. 5, the application of the dither signal results in an effect equivalent to processing in a converter with characteristic y_1, shown in Fig. 7.

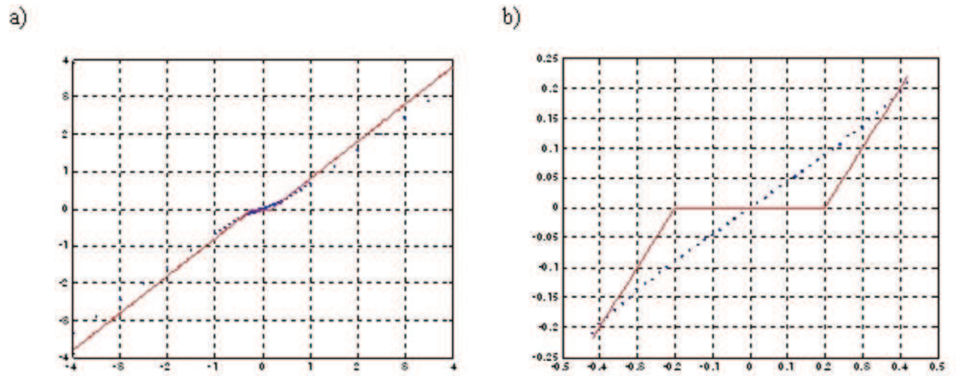


Fig. 7. Characteristic of a converter with dither: a) whole, b) fragment.

A modification of the static characteristic of a converter, resulting from DZ elimination, implies a change in the spectrum of the input signal. This change is shown in Fig. 8 for small values of input signals, not exceeding the dead zone (A= 0.18), as well as in Fig. 9 for values of signals exceeding the zone (A= 1). In every case identical parameters of spectral analysis were preserved. DZ elimination is the more effective the more similar the spectral structure of the output signal (y_1) becomes to the spectral structure of the prime input signal (s), which, except for the fundamental component, does not contain harmonics. The nonlinear distortion disappear.

4.1. Influence of the type of dither probability density function

The spectral structure of the signal at the output of a converter with modified characteristic does not depend on the PDF type. For dither with rectangular PDF and for dither with gaussian PDF differences are irrelevant if only the variances of both dithers are the same. This regularity occurs with processing of both small signals (not exceeding the DZ boundary, Fig. 8) and greater signals (exceeding DZ, Fig. 9). It can be noticed when comparing every two adjacent spectra in these figures.

4.2. Impact of dither variance

The effect of DZ elimination depends on the value of dither variance. In order to obtain the elimination effect, the relation of dither standard deviation to the boundary value of DZ should be greater than 1. On the other hand, because the use of the dither method reduces the dynamic range of the converter (margin for dither), this relation should be as small as possible. In the example considered in the paper this is the value of ca ($\sqrt{1.08}/0.2 \approx 5.2$ compare the last pairs of spectra in Fig. 8 and in Fig. 9).

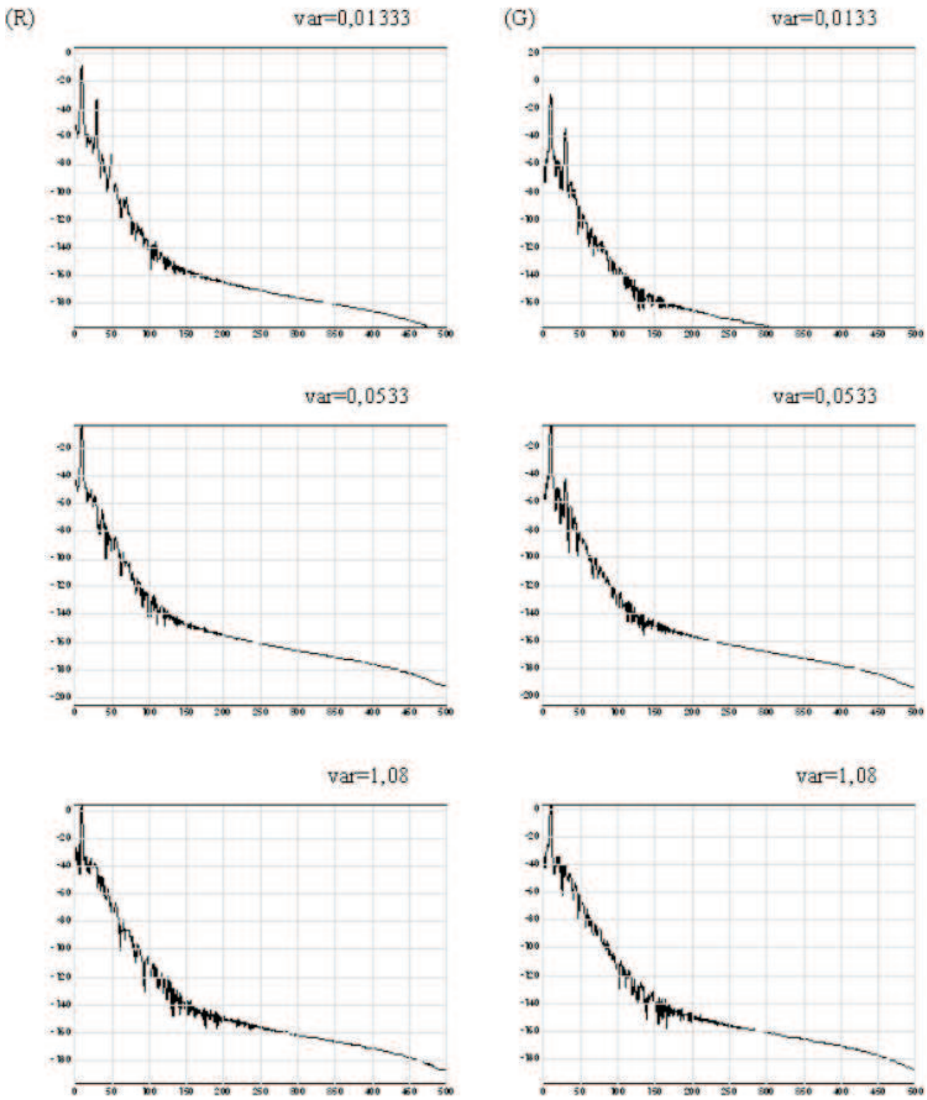


Fig. 8. Spectrum of the converter output signal, $-0.2 \leq \mathrm{DZ} \leq 0.2$ $\boldsymbol{A} = \mathbf{0.18}$: column (R) – dither with rectangular PDF, column (G) – dither with gaussian PDF.

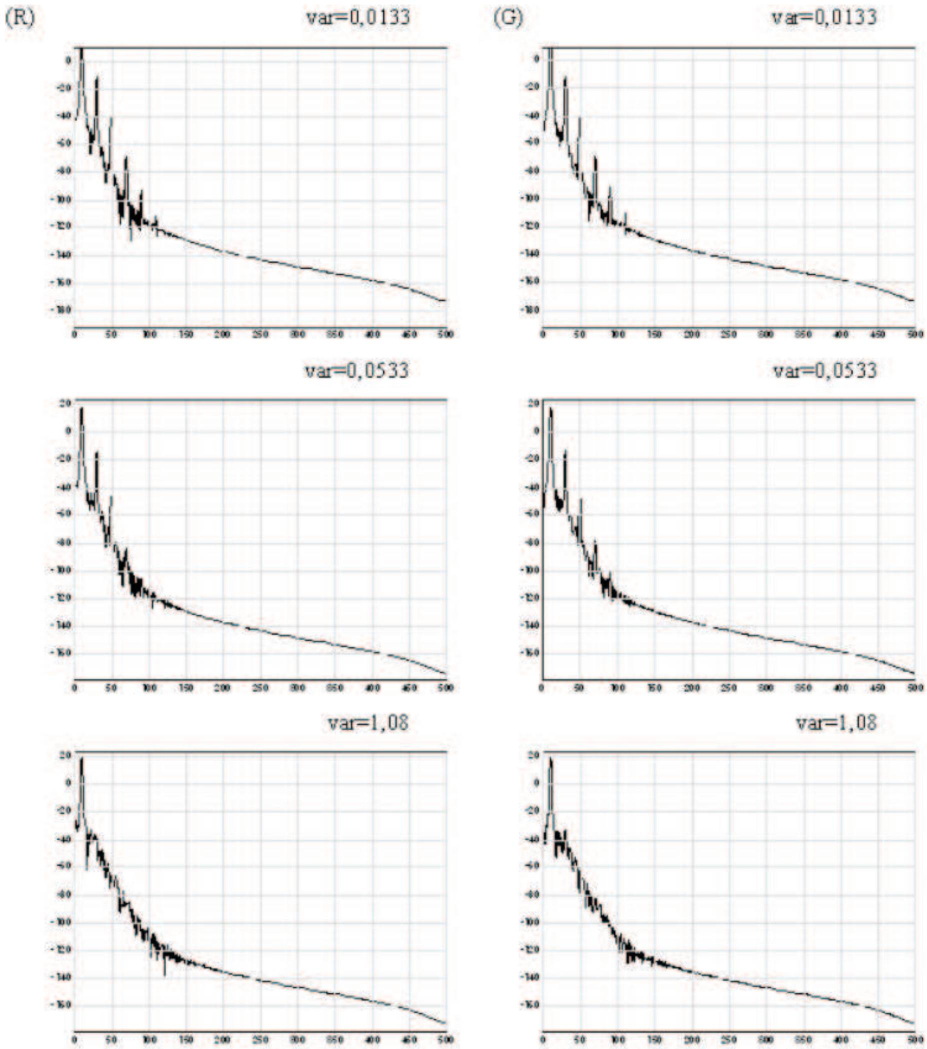


Fig. 9. Spectrum of the converter output signal, $-0.2 \leq \mathrm{DZ} \leq 0.2$ $\boldsymbol{A} = \mathbf{1}$: column (R) – dither with rectangular PDF, column (G) – dither with gaussian PDF.

5. ACCURACY OF A CONVERTER WITH MODIFIED CHARATERISTIC

Linearization of the characteristic with the use of the dither signal method requires two elements to be added to the converter: a dither generator and an AV/LPF system.

It is required from the generator to provide a signal characterized by stable parameters. In case of generating noise dither it is important that a zero average value and stability of this value is ensured.

Averaging or filtration are linear operations. The AV/LPF system introduces an amplitude and phase error. The influence of the system on the amplitude reduction can be noticed when comparing the output signals – in Fig. 3a (without system) and 6a (with system). The influence of the system on the phase shift can be noticed when comparing the input and output signal in Fig. 6a or in Fig. 6b. The converter itself does not introduce phase shift – compare the input and output signal in Fig. 3a.

6. CONCLUSION

The application of the dither signal processing method eliminates the DZ from a converter. It effectively removes nonlinearities in the output signal caused by the zone, i.e. removes nonlinear distortion of the processed signal. We can also apply the method to process signals with values wholly contained in the zone, which would be impossible in a converter without applying the method.

The effectiveness of the elimination method does not depend on the type of dither PDF (rectangular, gaussian), it depends, however, on dither variance. The relation of the standard deviation of dither to the boundary value of DZ should be a real number from the range (1,9).

Side-effects of the application of the method are:

- reduction in the converter dynamic range – the input signal plus dither must fall into the range of processing, which limits the acceptable (maximum) range of changes in the input signal itself;
- amplitude reduction and phase shift of the processed signal because of amplification error and phase error of the AV/LPF system, respectively. If we apply averaging (AV), it extends the time of processing.

In order to obtain the optimum result of linearization, a dither with suitable (in a given case) form of the PDF and a span/variance adjusted to the size of the DZ should be selected.

The dissimilarity of the method consists in the fact that in order to obtain the effect of elimination of DZ, we affect the input signal, intentionally making it altered.

The possibilities of applying the method do not depend on the operating rule of a converter/sensor or the technology used to produce them. It is important, however, to use the dither signal of the same type as the processed signal (electrical - electrical, mechanical - mechanical, optic - optic, etc.).

REFERENCES

1. Zakrzewski J.: *Measuring Sensors and Converters*. Wydawnictwo Politechniki Śląskiej, Gliwice, 2004 (in Polish).
2. Schuchman L.: "Dither signals and their effect on quantization noise". *IEEE Trans. Commun. Technol.,* vol. 12, 1964, pp. 162–165.

3. Jayant N., Rabiner L.: "The application of dither to the quantization of speech signals". *Bell Syst. Techn. J.*,vol. 51, 1972, pp. 1293–1304.
4. Gray R., Stockham T.: "Dithered quantizers". *IEEE Trans. Inform. Theory*, vol. 39, 1993, pp. 805–812.
5. Wagdy M., Goff M.: "Linearizing average transfer characteristics of ideal ADC's via analog and digital dither". *IEEE Trans. Instrum. Meas.,* vol. 43, 1994, pp. 146–150.
6. Carbone P., Petri D.: Performance of stochastic and deterministic dithered quantizers. *IEEE Trans. Instrum. Meas.*, vol. 49, 2000, pp. 337–340.
7. Adamo F., Attivissimo F., Giaquinto N., Savino M.: "Measuring the static characteristic of dithered A/D converters. Measurement", vol. 32, 2002, pp. 231–239.
8. Attivissimo F., Giaquinto N., Lancolla A., Savino M.: "Effects of midpoint linearization and nonsubtractive dithering in A/D convertrs". *Measurement*, vol. 40, 2007, pp. 537–544.
9. Zames G., Shneydor N.: "Dither in non-linear systems". *IEEE Trans. Autom. Contr.,* vol. 21, 1976, pp. 660–667.
10. Iannelli L., Johansson K., Jönsson U., Vasca F.: "Dither for smoothing relay feedback systems". *IEEE Trans. Circuis Syst. I, Fundam. Theiry Appl.*, vol. 50, 2003, pp. 1025–1035.
11. Iannelli L., Johansson K., Jönsson U., Vasca F.: "Averaging of nonsmooth systems using dither". *Automatica*, vol. 42, 2006, pp. 669–676.
12. Frasca R., Iannelli L., Vasca F.: "Dithered sliding-mode control for switched systems". *IEEE Trans. Circuis Syst. II, Express Briefs*, vol. 53, 2006, pp. 872–876.
13. Kajita T., Moon U., Temes G.: *"A two-chip interface for a MEMS accelerometer". IEEE Trans. Instrum. Meas.*, vol. 51, 2002, pp. 853–858.