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SOFT FAULT DIAGNOSIS IN ANALOG CIRCUIT BASED ON FUZZY AND DIRECTION VECTOR

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Abstract

A basic circuit theory of fault diagnosis for analog circuits with parameter tolerance is proposed in this paper. The approach uses the direction vector of voltage increment in test nodes as a fault signature for predefined faults. A linear equation is built to locate a faulty element. On the condition that the component tolerances are taken into account, the concepts of direction vector and fuzzy analysis method are combined together to analyze a parametric fault. Examples illustrate the proposed approach and show its effectiveness.

Keywords: analog circuit, fault diagnosis, fuzzy, direction vector.

1. Introduction

Since the 1970's, with the rapidly development of electric industry, testing and diagnosis play an important roles for the industry to efficiently continue moving forward. It is estimated in [1] that testing can account for up to 30% of the total manufacturing cost in 1993. In [2], it is reported that 95% of the test cost in mixed-signal circuits is expended in testing the analog parts. Therefore, the research on the diagnosis of analog circuits has become one of hot topics. Many methods have been proposed for fault diagnosis in analogue circuits [2-19]. It is popular to categorized those analog fault diagnosis techniques as simulation-before-test (SBT) and simulation-after-test (SAT) techniques [3-4]. The fault dictionary method [4,7], compared the circuit responses associated with predefined the fault values in the dictionary to locate the faults, is one of the most often used methods. In [8], an AC test was used to detect the fault. The test frequency was selected by using a comprehensive fault model and sensitivity information obtained from simulation on behavioral level. An algorithm in [9] based on fault model used DC stimuli and detects the catastrophic fault. However, they were not aimed at parametric faults caused by global process variations like mask misalignment

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and line width variations. In [10], single and multiple faults were detected based on circuit sensitivity computation. In [11], incremental sensitivity is used to detect both hard faults and soft faults and to analyze the observability of circuits. In [12], the test frequency was selected with constrained linear programming to be a math tool. In reference [13], by node-voltage sensitivity sequence dictionary, both hard and soft faults of any component are detected. However, if the CUT is in normal state with the influence of tolerance, the method is unable to identify the circuit's state. In [14], an approach to DC circuits uses a formula obtained on the basis of the Woodbury expression to identify fault parameter. It analyzes a circuit with nominal parameters and distinct excitations as well as measurements of some node voltages in a circuit with perturbed parameters. In reference [15], an approach using the linear-programming concept is proposed. Through checking the existence or absence of a feasible solution it is stated whether there is a fault or not in the circuit is stated. However, when an element changed heavily, the method is powerless. Reference [16] gives an approach to combined sensitivity analysis and fuzzy analysis to diagnose soft faults in linear analog circuits. By using membership function, the questions of test node selection and fault diagnosis are handled at the same time. In [17], a comparison between different techniques in the field of analog circuits is made, with applications. Reference [18] focuses the attention on the sensitivity analysis from the diagnostic point of view. In [19], authors take into account the optimal selection of the test frequencies with the aim to localize the faulty element in the analog CUT.

In this paper, a basic circuit theory and a novel analysis method for analog circuits with parameter tolerance are proposed. A relationship between the circuit parameters and the measured voltage increments is established. An algorithm is presented so that the theory can be applied to a circuit under test (CUT). For diagnosis of parametric faults in a CUT with tolerance, a new diagnosis method which combines both direction vector and fuzzy analysis is developed.

The paper is organized as follows. Section 2 presents some basic mathematic definitions of fuzzy and fuzzy math expression in fault diagnosis. Section 3 provides a diagnosis methodology, including the principle of direction vector method, the fuzzy math expression in fault diagnosis with tolerance and determination of membership function. In Section 4, two experimental results and comparison with another method are given to show the effectiveness of the proposed method. Conclusions are summarized in Section 5.

2. The Fuzzy Definition for Fault Diagnosis

The fuzzy set concept was originated by Zadeh [20-22]. Instead of taking on only two values 0 or 1 depending on "included in" or "not included in" the set, the basic idea involves defining a membership function for each element of the referential set. The membership function takes its value in the interval [0,1], depending on the degree of belonging to the set.

2.1. Fuzzy Definition

For clarity, some of the definitions are repeated in the following discussions.
Definition 1: Let E be a referential set and x be an element of E. Then, a fuzzy subset A of E will be defined by its membership function, which means the element of E belongs to A with the level located in [0, 1].

$$\forall x \in E : \quad \mu_A(x) \in [0, 1]. \tag{1}$$

Two major operations which will form fuzzy sets as a lattice structure are defined as follows.
Definition 2: Let E be a set, x be an element of E, and *mu* (x) be its membership function. Let A and B be two fuzzy subsets of E.
The union of subsets A and B is

$$\mu_{A\cup B}(x) = Max[\mu_A(x), \mu_B(x)]. \tag{2}$$

The intersection of subsets A and B is

$$\mu_{A\cap B}\,(x) = Min\,[\mu_A\,(x),\, \mu_B\,(x)]. \tag{3}$$

Since [0, 1] is a complete lattice, we can define intersections and unions of arbitrary families

$$\text{Intersect}: \quad \underset{i\in I}{I\mu_{Ai}}(x) = \inf_{i\in I} \mu_{Ai}(x). \tag{4}$$

$$\text{Unions}: \quad \underset{i\in I}{U\mu_{Ai}}\,(x) = \sup_{i\in I}\, \mu_{Ai}\,(x). \tag{5}$$

2.2. Membership Function

To diagnose a fault in the CUT, choosing a reasonable membership function is very important. Although how to determine a membership function is still a question for all researchers in the world, according to fuzzy math there are some available functions for membership functions in real domain, such as rectangle function, triangle function, Gaussian function, Cauchy function and so on. Among those functions, Gaussian functions are used morew frequently, which can best present a fuzzy set of faults and have small computation-time consumption. Formula (6) presents Gaussian function.

$$\mu_A\,(x; a, k) = \exp\left(-\left(\frac{x-a}{\delta}\right)^2\right) = \exp\left(-k\,(x-a)^2\right), \tag{6}$$

where a is the central point and k is the deviation ($k > 0$).

2.3. Fuzzy Math Expression in Fault Diagnosis

Suppose that the fault set is $F = \{F_0, F_1, F_2, ..., F_p\}$ $(1 \leq p \leq n)$, where n is the number of elements in CUT and F_0 denotes the normal state, which means there is no faulty element in CUT. $A = \{A_1, A_2, ..., A_m\}$ is the test node set and m is the number of test nodes.

In CUT, when there is a fault F_i $(0 \leq i \leq p)$, the voltage increment in each test node can be measured as $[\Delta u_{i1}, \Delta u_{i2}, ..., \Delta u_{im}]$. Then, the direction vector in each test node is calculated as

$$\overrightarrow{\Delta u_{ij}} = \Delta u_{ij} \Bigg/ \sqrt{\sum_{j=1}^{m} \left(\Delta u_{ij}\right)^2} \, . \tag{7}$$

Therefore, for fault F_i $(0 \leq i \leq p)$, its symbol direction vector $\overrightarrow{\Delta_{u_i}} = \left(\overrightarrow{\Delta u_{i1}}, \overrightarrow{\Delta u_{i2}}, ..., \overrightarrow{\Delta u_{im}}\right)^T$ is a $m \times 1$ dimensional vector. When there is a fault in the CUT, after calculating the direction vector of measured values in all test nodes, the fault diagnosis of multi-node is to determine in which state the CUT is according to the fuzzy set.

From the definition of intersection of subsets and minimum degree of membership criterion, the degree of the state of CUT subordinated to F_i is defined as the intersection of each vector component membership degree u_{Fij} $(\overrightarrow{\Delta u_{ij}})$. So according to the definition of the intersection function

$$u_{Fi}\left(\overrightarrow{\Delta u}\right) = Min\left[u_{Fi1}(\overrightarrow{\Delta u_1}), u_{Fi2}(\overrightarrow{\Delta u_2}), ..., u_{Fim}\left(\overrightarrow{\Delta u_m}\right)\right]. \tag{8}$$

Then, from the definition of union of subsets and maximum degree of membership criterion, if u_{Fi} $\left(\overrightarrow{\Delta u}\right)$ satisfies u_{Fi} $\left(\overrightarrow{\Delta u}\right) = Max\left\{u_{F1}\left(\overrightarrow{\Delta u}\right), U_{F2}\left(\overrightarrow{\Delta u}\right), ..., u_{Fp}\left(\overrightarrow{\Delta u}\right)\right\}$, it can be deemed that the measured direction vector $\left(\overrightarrow{\Delta u}\right)$ is subordinate to F_i, and the current state of the CUT is more similar to fault state F_i.

For multi-node diagnosis, if the values of membership functions at each nodes for fault are close to 1, from (8) the value of the intersection function for fault is close to 1 and it means the current fault code is corresponding to the fault code of fault if F_i; if the value of any membership function is close to 0, the value of the intersection function for fault is close to 0 and it means the current fault code is not corresponding to the fault code of fault F_i. To be pointed out, the result of membership function may be 1 and it only means the current state and fault state have the same character and are the most similar, but possibly the CUT is not in a fault state.

3. Diagnostic methodology

In this section, the fundamental theories of our diagnosis approach, fuzzy math expression for diagnosis, and membership function are discussed.

3.1. Principle of the Direction Vector Method for the Diagnosis of Faulty Element

Let us suppose that a CUT with n elements and m test nodes ($1 \le m \le n$) consists of linear resistors, inductors, capacitors, and voltage-controlled current sources, all with nominal parameters and independent current sources.

When an element connected to nodes k and q is faulty and its admittance is perturbed from Y to $Y + \Delta Y$, the measured node voltages of accessible test nodes change from U to $U + \Delta U$, where $\Delta U = [\Delta u_1, ..., \Delta u_m]$. In [15], it is shown that the deviation of the i th-node voltage is given by

$$\Delta u_i = -(z_{ik} - z_{iq} \frac{\Delta Y}{1 + \delta \Delta Y} (u_k - u_q), (i = 1, ..., m), \tag{9}$$

where $z_{ij}(i, j = 1, ..., m)$ are elements of the node impedance matrix and $\delta = z_{kk} - z_{kq} + z_{qq}$. That is, due to the parameter perturbations of elements, the voltage deviation at the test point is a linear combination of the voltages across those elements in CUT. Therefore, according to [15], it can be easily obtained.

$$\frac{\Delta u_i}{\left(\sum_{j=1}^{m} \Delta u_j^2 \right)^{1/2}} = \frac{z_{ix}}{\left(\sum_{j=1}^{m} z_{jx}^2 \right)^{1/2}}. \tag{10}$$

So, it can be obtained that

$$\overrightarrow{\Delta u_i} = \frac{\Delta u_i}{\left(\sum_{j=1}^{m} \Delta u_j^2 \right)^{1/2}} = \frac{z_{ix}}{\left(\sum_{j=1}^{m} z_{jx}^2 \right)^{1/2}}. \tag{11}$$

Equation (11) shows that $\overrightarrow{\Delta u_i}$ is a constant dependent on the position and the nominal parameter of the element in CUT. But it is independent of the parameter perturbation ΔY. Therefore, to m test nodes, $\overrightarrow{\Delta u} = \left[\overrightarrow{\Delta u_1}, \overrightarrow{\Delta u_2}, ..., \overrightarrow{\Delta u_m}\right]$ can be defined as the symbol direction vector of a single fault in the CUT.

So, in a linear analog circuit, when a fault occurs in any component, no matter how great the change magnitude of the faulty parameter is, the direction vector of node-voltage increment caused by the faulty component is invariant. And, if all the faults in one component are regarded as one-kind fault, the size of the fault dictionary is exactly equal to the number of the components in CUT and all symbol direction vector of elements can establish a $m \times n$ dimensional symbol matrix.

3.2. The Diagnosis Process for Soft Fault with Tolerance Based on Fuzzy Math and Direction Vector

In practice, due to the element parameter tolerance, when a fault occurs in the CUT, the real parameters of other components are randomly changed in their tolerance range and the measured voltage increment in each node is within a little range. Therefore, the calculated direction component in each test node is correspondingly changed around its nominal value. In order to locate the faulty element under the influence of tolerance, the diagnosis steps based on fuzzy math and direction vector are as follows:
$x = (x_1, x_2, ..., x_m)^T$ expresses the tested fault vector, where m are the test nodes.

- Step 1 Define the referenced vectors of faults in fault set $A = \left[u_{F1}, u_{F2}, ..., u_{Fp}\right]$, where $F_{j(j=1,2,...,p)}$ is the predefined fault state in fault set and $u_{Fj} = \left[\overrightarrow{\Delta u_{1j}}, \overrightarrow{\Delta u_{2j}}, ..., \overrightarrow{\Delta u_{mj}}\right]^T$. Let $i = 1,\ j = 1$.
- Step 2 Calculate the relation degree between the component $x_{i\ (i=1,2,\cdots,m)}$ of the tested direction vector of $\overrightarrow{\Delta u}$ and the component $\overrightarrow{\Delta u_{ij}}\ _{(i=1,2,...,m)}$ of referenced vectors in A referred to (8). Then, $i = i + 1$.
- Step 3 If $i < m$, go to step 2;
  If $i = m$, calculate the intersection relation degree of the tested vector to supposed fault F_j referred to (4). Then, let $j = j + 1,\ i = 1$.
- Step 4 If $j = p$, then go to step 5, else return Step 2.
- Step 5 Calculate the union relation degree of all u_{Fj}. Then, obtain the final result referred to (5).

3.3. Determination of Membership Function

From 3.1. it is known that in each fault state there is a symbol direction vector and the direction component for each test node is constant. So, in deciding on the membership function, the component of the symbol direction vector can be regarded as the center parameter a in membership function. Parameter k of membership function is estimated from the fixed width approach. As in 3.2, due to the element parameter tolerance, the calculated direction component changes around its nominal value. In each test node, if we think that the value of the Gaussian function in (6) is equal to or greater than 0.9, the calculated direction component is close to the component of symbol direction vector. Then, suppose parameter a expands a fixed width δ at each side and the value of the membership function in each node is 0.9. The parameter k can be calculated from $k = -\dfrac{\ln 0.9}{(u_r - a)^2} = -\dfrac{\text{in } 0.9}{\delta^2}$, where is the measured direction vector component in test node when the CUT is under fault state. If set $\delta^2 = (u_r - a)^2 = 0.05v^2$, then $k \approx 2.107v^2$.

According to fuzzy math, the membership function can be divided into three segments: 1-segment, abrupt-segment and 0-segment. In 1-segment, the values of the membership function are equal or close to 1, and the nominal value is the center. Abrupt-segment is a transitional segment which presents a sudden change of the membership function. Out of abrupt-segment, it enters the 0-segment, where the similarity between the current state and the fault state is very low and which means the CUT is in another fault state.

4. Experiment Results and Discussion

In this section, experiments are made to prove the effectiveness of the proposed method. All the simulation and the calculations were performed with the PSPICE program [23] and MATLAB [24] in a PC with Intel T2080@1.73GHz and 504 MB.

4.1. Fault Diagnosis without Tolerance

An example of a current benchmark circuit in [25] is shown in Fig. 1. Here, the following values have been taken: $R_1=R_2=R_3=R_4=R_5=10k\Omega$, $C_1=C_2=20nF$, $R_6=3k\Omega$, and $R_7=7k\Omega$. According to the approach presented in the previous section, in order to diagnose all faults in resistance, a 1Vdc DC voltage source is placed as input of the CUT. The faults in capacitors are diagnosed using the same method when the stimulus of CUT is changed to a 500 Hz, 1Vac AC voltage source. All nodes, except for the input node, in the CUT are chosen as test nodes. The CUT is tested with bias points analysis obtained by inducing single faults to the circuit in the component value from the nominal value. A linear matrix equation $A_{m\times n}x = b$ is built and solved to diagnose the CUT.

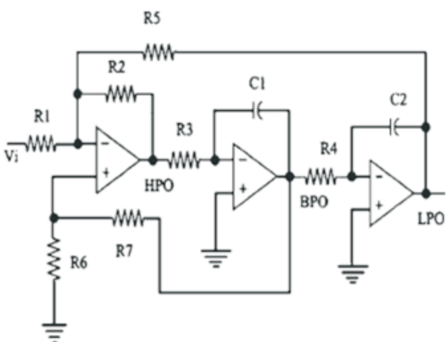

Fig. 1. Benchmark circuit (Continuous-Time Sate-Variable Filter).

Seven suspicious failure conditions of the components are defined as shown in Table 1(A) and Table 1(B). The diagnosis results are given at the same time.

The solution of the linear equation shows the position of a faulty element. It means that if the *i* th element is faulty, the *i* th component in the solution vector is equal to or close to one and other components are zero. From the calculated results in Table 1, in all solution vectors for preset faults, it can be seen that all soft fault can be located correctly, which means the method we assumed above is valid.

Table 1(A). Diagnosis results with DC Input.

	R_1=30K	R_2=5K	R_3=20K	R_4=30K	R_5=1K	R_6=0.5K	**R_7=20K**
R_1	**9.9996e-01**	4.1678e-04	-1.2811e-03	8.8882e-04	1.9419 e-04	-1.5258e-04	**4.4331 e-05**
R_2	5.8821e-09	**-9.9958e-01**	-1.2341e-09	3.7252e-09	2.0010 e-09	-1.8626e-09	**-4.4332 e-05**
R_3	3.0434e-12	-0.0000e-01	**1.0000 e-00**	-7.0030e-11	-1.5139 e-11	1.1820 e-11	**-3.4558 e-12**
R_4	2.0326 e-12	-0.0000e-01	6.6770 e-11	**9.9999 e-01**	-1.0121e-11	7.7300 e-12	**-2.3104 e-12**
R_5	-3.9014e-05	4.1678 e-04	-1.2811e-03	8.9263 e-04	**-9.9981 e-01**	-1.6021e-04	**4.4331 e-05**
R_6	2.9553e-10	8.7169 e-03	1.0486 e-08	-8.1781e-09	-1.4234e-09	**-9.9999 e-01**	**-9.2734 e-04**
R_7	**2.9826 e-10**	**8.7169 e-03**	**1.0575 e-08**	**-8.1781e-09**	**-1.437 e-09**	**1.5134 e-09**	9.9907-01

V_i=1Vdc

Table 1(B). Diagnosis Results with AC Input.

	C1=40nF	C2=40nF
R1	-0.0953 - 0.2165i	0.0398 + 0.1083i
R2	-0.1376 - 0.2548i	-0.1704 + 0.0339i
R3	-0.3076 + 0.0487i	-0.0557 - 0.0471i
R4	-0.0826 - 0.031i	-0.3445 + 0.013i
R5	-0.0423 - 0.0383i	-0.2102 - 0.0744i
R6	0.107 + 0.0035i	-0.1308 - 0.023i
R7	0.107 + 0.0035i	-0.1308 - 0.023i
C1	0.6924 + 0.0487i	-0.0557 - 0.0471i
C2	-0.0826 - 0.031i	0.6555 + 0.013i

V_i=1Vac(f=500Hz)

4.2. Fault Diagnosis with Tolerance

Let us consider the transistor circuit shown in Fig. 2, where nodes 1,2,5,6,8-10 are accessible nodes for measurement. The nominal resistance values are shown in Fig.

2 and the tolerance of any resistance is assumed as 5% of the nominal value. If the parameter perturbation of each element is beyond its tolerance range, it is thought to be faulty.

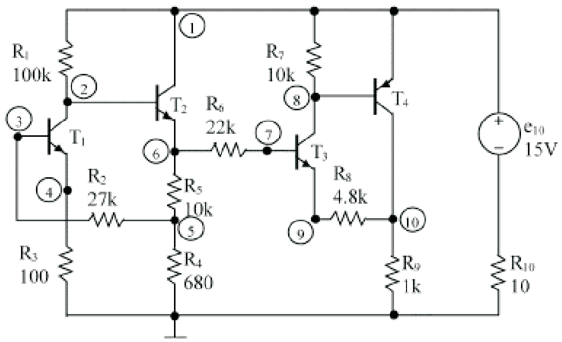

Fig. 2. A DC circuit.

The referenced direction vector values of single faults (without tolerance) can be calculated according to (11), and are shown in Table 2.

Table 2. The referenced direction vectors of single faults (without tolerance).

F1	(0.00550,-0.51009,-0.03211,-0.50798,0.00796,-0.50354,-0.47654)
F2	(-0.00517,0.51022,0.03241,0.50813,-0.00756,0.50356,0.47621)
F3	(-0.00518,0.51018,0.03240,0.50809,-0.00759,0.50355,0.47630)
F4	(0.00523,-0.51004,0.00609,-0.50751,0.00833,-0.50358,-0.47809)
F5	(-0.00442,0.51001,-0.01891,0.50989,-0.00640,0.50422,0.47459)
F6	(0.00687,0.00095,0.00006,0.00096,0.01036,-0.72639,-0.68717)
F7	(-0.00996,-0.00114,-0.00006,-0.00109,-0.01585,0.08162,0.99649)
F8	(0.00999,0.00137,0.00008,0.00138,0.01523,0.00622,-0.99981)
F9	(0.17011,0.02326,0.00148,0.02349,0.27161,0.10095,0.94128)
F10	(-0.69383,-0.09482,-0.00603,-0.09555,-0.69375,-0.09809,-0.09764)

According to the values in Table 2, the fault set here can be defined as $F = \{F_1, F_2, ..., F_{10}\}$. Certainly, when the number of tested nodes are decreased, the fault set should be redefined.

For single fault diagnosis with tolerance in the CUT, the Gaussian distribution function is used as the membership function and the similarity degree of the tested fault to F_i in fault set is calculated as

$$u_{Fi} = Min\left(\exp\left\{ -2.107 \times \left(\overrightarrow{\Delta u_J} - a_{ij} \right)^2 \right\} \right), \tag{12}$$

where a_{ij} represents the j th sub-vector value of F_i 's referenced direction vector and $\overrightarrow{\Delta u_j}$ represents the j th sub-vector value of the measured direction vector.

4.2.1. Diagnosis with different faults in one element

Since variations in analog component parameters, which result in output variations, are random in nature, they are modeled by conducting a series of Monte-Carlo simulation runs. In each of these runs, component parameters are randomly varied by a certain (user-defined) percentage around their nominal values. Here, an element in the CUT is randomly chosen as the faulty element. 50 Monte-Carlo simulations with DEV=5% are invoked. Supposing that soft faults occur in R_4 and the parameter of R_4 changes to 720 Ω (slightly out of tolerance range), 1360 Ω (double of the nominal value) and 200k Ω.

Table 3. Part calculated direction vectors of Monte-Carlo simulations (with tolerance σ=5%) with faults in R4.

R4=720	(-0.09681,-0.50525,0.00561,-0.50321,-0.09443,-0.49929,-0.47317)
	(-0.06558,-0.50537,0.00375,-0.50375,-0.06375,-0.50093,-0.48118)
	(-0.19625,-0.50896,0.00235,-0.50565,-0.19398,-0.49752,-0.40199)
	(-0.10754,-0.50476,0.00649,-0.50277,-0.10644,-0.50268,-0.46563)
	(-0.09641,-0.49582,0.00594,-0.49376,-0.09513,-0.49167,-0.50025)
R4=1360	(-0.00085,-0.51004,0.00605,-0.50752,0.00223,-0.50361,-0.47813)
	(-0.00087,-0.50978,0.00561,-0.50731,0.00216,-0.50354,-0.47872)
	(-0.00167,-0.51101,0.00582,-0.50842,0.00139,-0.50428,-0.47544)
	(-0.00150,-0.51010,0.00622,-0.50759,0.00145,-0.50398,-0.47761)
	(-0.00064,-0.50947,0.00610,-0.50695,0.00234,-0.50313,-0.47986)
R4=200K	(0.00504,-0.51587,0.00655,-0.50877,0.03121,-0.50368,-0.46935)
	(0.00504,-0.51597,0.00623,-0.50885,0.03191,-0.50371,-0.46908)
	(0.00503,-0.51592,0.00636,-0.50881,0.03050,-0.50374,-0.46923)
	(0.00503,-0.51582,0.00670,-0.50873,0.03088,-0.50374,-0.46941)
	(0.00505,-0.51584,0.00658,-0.50874,0.03123,-0.50364,-0.46946)

In Table 3, part of calculated direction vectors when R_4 equals different values is given.

For example, when R_4 = 1360 Ω, one of the measured direction vectors is $\overrightarrow{\Delta u}$ = (-0.00087, -0.50978, 0.00561, -0.50731, 0.00216, -0.50354, -0.47872) and the similarity degree of the fault to each F_i in the fault set is:

u_{F1} = 0.99991Λ1Λ0.99701Λ1Λ0.99993Λ1Λ0.99999 = 0.99701

u_{F2} = 0.99996Λ0.11168Λ0.99849Λ0.11389Λ0.99980Λ0.11800Λ0.14641 =0.11168

u_{F3} = 0.99996Λ0.11170Λ0.99849Λ0.11391Λ0.99980 Λ0.11801Λ0.14636 = 0.11170

u_{F4} = 0.99992Λ1Λ1Λ1Λ0.99992Λ1Λ1 = 0.99992

u_{F5} = 0.99997Λ0.11178Λ0.99873Λ0.11303Λ0.99984Λ0.11767Λ0.14737=0.11178

u_{F6} = 0.99987Λ0.57719Λ0.99994Λ0.58023Λ0.99986Λ0.90065Λ0.91252 = 0.57719

u_{F7} = 0.99983Λ0.57979Λ0.99993Λ0.58279Λ0.99930 Λ0.48603Λ0.01020 = 0.01020

u_{F8} = 0.99975Λ0.57666Λ0.99994Λ0.57971Λ0.99964 Λ0.57838Λ0.56432 = 0.56432

u_{F9} = 0.94026Λ0.54955Λ0.99996Λ0.55230Λ0.85815Λ0.46305Λ0.01428 = 0.01428

So, according to (5), $u_{F4} = \sup_{i=1}^{10} u_{Fi}$, the tested fault is F_4 in the fault set. Therefore, R_4 is the faulty component in the CUT.

Similarly, all faults in R_4 can be diagnosed through Monte-Carlo simulations can be diagnosed as above. When R_4 equals 720 Ω, because the element value is just slightly out the tolerance range little (0.88%), there are erroneous judgments in diagnosis results. When equals 1360 Ω and 200k Ω, almost all diagnoses can be correctly located to the real faulty element, which means that the magnitude of faults in a faulty element have no influence on the diagnosis results based on the method proposed in this paper.

4.2.2. Diagnosis with faults in a different element

Three cases with the following faults were studied.

- Case 1: element R_5 is faulty and R_5 = 11.5kΩ. All the other parameter of the CUT are R_1 = 101kΩ, R_2 = 26.5k R_3 == 103Ω, R_4 = 650Ω, R_6 = 22.5kΩ, R_7 = 10.05kΩ, R_8 = 4.75kΩ, R_9 = 1.01kΩ and R_{10} = 10Ω. ×
- Case 2: element R_1 is faulty and R_1 = 200kΩ. R_5 = 9.95kΩ, the other parameters of the CUT remain as in Case1.
- Case 3: element R_8 is faulty and R_8 = 9.6kΩ. R_5 = 9.95kΩ, the other parameters of the CUT remain as in Case1.

According to the diagnosis method proposed in the paper, the similarity between exampled fault and predefined fault set is calculated. Table 4 gives the intersection results of the above three exampled faults subordinated to the predefined faults in the fault set. From the maximum similar degree values in each case, the faulty element is located correctly. The results show that under the influence of parameter tolerance, the proposed method in this paper still gives the correct fault types for all the sampled faults.

Table 4. The intersection results of exampled faults to predefined faults.

	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
case 1	0.11221	0.99668	0.99668	0.11223	0.99972	0.04145	0.56998	0.009912	0.60850	0.36738
case 2	0.99062	0.10892	0.10894	0.97700	0.10902	0.56998	0.01254	0.52293	0.01742	0.35090
case 3	0.14511	0.07396	0.07393	0.14514	0.07453	0.05550	0.00365	0.98502	0.00530	0.35127

4.3. Comparison with other method

In reference [16], an example of a linear resistive analog circuit is given as Fig. 3, where I_s=1A, R_1 =R_2 = R_4 =R_5 =1 Ω, R_3=0.5 Ω. The tolerance of each element is set as 10%. The candidate test node set is {①, ②, ③,}.

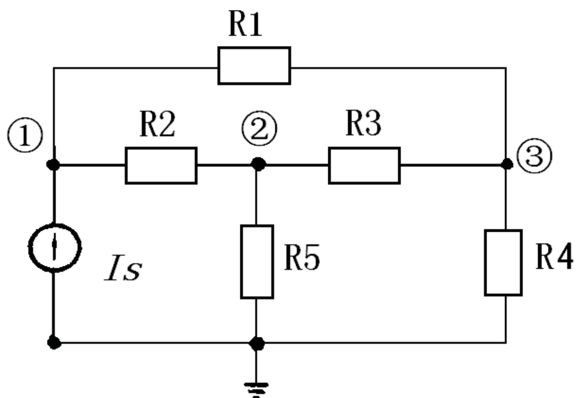

Fig. 3. A linear resistive analog circuit as an example in [16].

In reference [16], each fault state is defined as the faulty element has a fixed value, which make its fault set infinite. And, according to reference [16], double or half of the nominal sensitivity ratio is the choice to calculate parameter *k* in the membership function, which cannot show clearly whether an element value is out of its tolerance range. Therefore, when the faulty element's value changes heavily, in some condition, incorrect fault location cannot be unavoidable.

Here, the method in [16] and the method proposed in this paper are used to diagnose the parameter fault in the CUT shown in Fig. 3. Table 5 gives partial diagnosis results of comparison of the two methods. All the measured voltage increment in Table 5 are the results of Monte-Carlo simulation with components changing within their tolerance (σ=10%) under the condition of Gaussian distribution.

Table 5. Partly diagnosis results of two methods.

R1=0.5 tolerance=10%			R1=100 tolerance=10%		
voltage increment in nodes $(\Delta u_1, \Delta u_2, \Delta u_3)$	Result1	Result2	voltage increment in nodes $(\Delta u_1, \Delta u_2, \Delta u_3)$	Result1	Result2
(0.714286,0.285714,0.428571)	F_1	F_1	(1.385826,0.204724,0.397637)	F_1	F_1
(0.698747,0.269076,0.411181)	F_1	F_1	(1.392674,0.193993,0.377660)	F_3	F_1
(0.738339,0.281272,0.441687)	F_1	F_1	(1.455955,0.205426,0.389226)	F_2	F_1
(0.718372,0.292945,0.438510)	F_1	F_1	(1.443553,0.216722,0.409166)	F_1	F_1
(0.701264,0.288671,0.416817)	F_2	F_1	(1.299740,0.206992,0.391689)	F_2	F_1
(0.716818,0.297120,0.423785)	F_2	F_1	(1.444034,0.216580,0.419562)	F_5	F_1
(0.700610,0.278780,0.408992)	F_1	F_1	(1.418745,0.201877,0.394843)	F_2	F_1
(0.720530,0.271765,0.423193)	F_1	F_1	(1.444124,0.190715,0.388195)	F_1	F_1
(0.737726,0.308202,0.449402)	F_1	F_1	(1.391149,0.225704,0.418379)	F_3	F_1
(0.727122,0.285106,0.438630)	F_1	F_1	(1.404477,0.211777,0.386834)	F_3	F_1

Result 1: diagnosis result using the method in reference [16].

Result 2: diagnosis result using the method introduced in this paper.

From the diagnosis results, firstly, when the faulty magnitude is not very high (50%), the diagnosis ratio in this paper is improved compared to [16]. Secondly, because of the limitation of a fault set defined in [16], when an element changes heavily, the fault diagnosis ratio is low. However, under the fault set defined in this paper, the question does not appear. Therefore, the two questions in [16] mentioned above are solved in this paper.

5. Conclusions

A new approach to locate a single soft fault in an analog circuit is presented. In this paper, for the diagnosis of an analog circuit fault, the fuzzy math and the direction vector of the voltage increment are combined together. A linear equation whose coefficient matrix is composed of the direction vector of the voltage increment is built to identify a parametric fault of an element. From the solution of the equation, the faulty element is located. When the tolerance influence is considered, a soft fault diagnosis strategy is presented using fuzzy math. The membership function decided by a fixed width approach is used to identify the faulty state. The given examples show that the method proposed is effective and the diagnosis accuracy is high. Compared to other methods, the predefined fault set is simplified and the diagnosis ratio is increased.

In addition, the approach is proven to be valuable in the diagnosis of multi-faults (two or three faults) in an analog circuit, which will be presented in another paper.

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