

## RESPONSE TIME OF AIR GAUGES WITH DIFFERENT VOLUMES OF THE MEASURING CHAMBERS

Mirosław Rucki<sup>1)</sup>, Branimir Barisic<sup>2)</sup>

1) Poznan University of Technology, Division of Metrology and Measurement System, Institute of Mechanical Technology, Piotrowo 3, 60-965 Poznań, Poland (✉mirosław.rucki@put.poznan.pl, +48 61 665 3568)

2) University of Rijeka, Technical Faculty, Vukovarska 58, 51000 Rijeka, Croatia

### Abstract

In the paper, investigations on the dynamic properties of air gauges are presented. The measurement of the amplitudes of back-pressure  $p_k$ , dependent on the input signal angular frequency  $\omega$  for the group of air gauges with various parameters has been performed. To evaluate the influence of the measuring chamber volume on the dynamical properties, a set of the model chambers with different dimensions has been prepared. The obtained results underwent a repeatability test in order to evaluate the uncertainty of time constant determination. Additionally, the impact of the feeding pipe was examined. Examinations of the time constant of typical back-pressure air gauge led to the conclusion that it depends also on the actual pressure and that it may differ between the beginning and the end of the measuring range.

Keywords: air gauging, frequency characteristics, dynamical properties, geometrical metrology.

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### 1. Introduction

Air gauges have been used as length measuring devices for decades. They are widely applied in many branches of Mechanical Engineering, especially for in-process control. The main merits of air gauges are: the possibility of non-contact measurement, high accuracy, high sensitivity, high immunity to the influence of external conditions and adaptability to a wide range of different measuring applications [1], [2]. When the measurement is performed during a technological operation (in-process control), or continual data on the profile is to be collected (like in non-contact pneumatic devices for roundness and cylindricity measurement), the dynamic properties of the air gauge are of high importance. Dynamic variables are time- or space-dependent in both their magnitude and frequency content.

When periodic inputs are applied to a first-order system, the frequency of the input has an important influence on the measuring system's response and affects the output signal [3]. When the input signal forms a simple periodic function,  $F(t) = A \cdot \sin \omega t$ , and the initial conditions are  $y(0) = y_0$ , then the function could be written as

$$T\dot{y} + y = KA \sin \omega t, \quad (1)$$

where:

$T$  – time constant,

$K$  – static sensitivity,

$A$  – amplitude,

$\omega = 2\pi f$  [rad/s] – rotational speed,

$f$  [Hz] – frequency.

The general solution to this differential equation yields the measuring system output signal, the time response to the applied input,  $y(t)$  [3]

$$y(t) = Ce^{-t/T} + \frac{KA}{\sqrt{1+(\omega T)^2}} \sin(\omega t - \tan^{-1} \omega T), \quad (2)$$

where  $C$  is a constant, dependent on the exact value of  $y_0$ .

The amplitude of the steady response depends on the value of the applied frequency  $f$ .

Most of the papers dedicated to pneumatic measurement methods were published a few decades ago. They provided some methods to calculate the dynamical properties of one-cascade back-pressure air gauges, like [4], [5] or [6]. However, their investigations dealt with air gauges with large volumes of measuring chambers with additional dead volumes of pressure transducers which caused a time constant of a few seconds. More recent works were dedicated to smaller chambers combined with piezoresistive pressure transducers, like [7] or [8], but they did provide neither wide experimental background nor recommendations on the possible improvement of dynamical properties of the air gauges.

## 2. Experimental setup

In order to investigate the behavior of the air gauge with sinusoidal input signal, the following equipment has been used.

A model of the air gauge with certain inlet ( $d_w$ ) and outlet ( $d_p$ ) nozzles was prepared for the investigations. Using the equipment described in the paper [9], static metrological characteristics were determined, including measuring range  $z_p$  and sensitivity  $K$ . Because in some cases the outer diameter ( $d_c$ ) of the measuring nozzle could have an impact on the metrological properties of the air gauge, different relations  $d_c/d_p$  were taken into consideration. The Table 1 presents the parameters of examined air gauges.

Table 1. Dimensions and characteristics of the examined air gauges.

<i>Index</i>	$d_p$ [mm]	$d_c/d_p$ [-]	$d_w$ [mm]	$K$ [kPa/ $\mu$ m]	$z_p$ [ $\mu$ m]
D2	1.2	1.5	1.200	0.15	170
D3	1.2	1.5	0.840	0.39	110
D5	1.2	1.5	0.625	0.88	74
D6	1.2	3.0	1.200	0.19	114
D7	1.2	3.0	0.840	0.52	108
D8	1.2	3.0	0.625	0.77	78

It is seen from Table 1 that the static properties of the air gauges are quite different just because of different outer diameter  $d_c$ . For example, comparing the sets indexed as D2 and D6, the change of  $d_c$  causes more than a 20% change in sensitivity  $K$  and measuring range  $z_p$ . It should be expected that it may have an impact also on the dynamic properties of the air gauge.

In fact, none of actually used methods of dynamical calculation is able to take into consideration the nozzle head width  $d_c/d_p$ . The most commonly used method described in [10] is based on the time of recovering of the pressure  $p_k$  inside the measuring chamber down to a certain value which is fed after some air left it through the larger measuring slot. The formula describing the process depends on the measuring chamber volume and the air outlet surface, and it is as follows for the higher feeding pressures [10]

$$T = \frac{v_k \rho_0 \tau_0 R g}{p_0} \frac{1}{\left( \frac{\partial G_2}{\partial w_{2sr}} \frac{\partial w_{1sr}}{\partial p_k} \right) - \left( \frac{\partial G_1}{\partial w_{1sr}} \frac{\partial w_{1sr}}{\partial p_k} \right)} \quad (3)$$

where:

$v_k$  – volume of measuring chamber,

$\rho_0$  – mass density,

$\tau_0$  – temperature,

$R$  – gas constant,

$g$  – acceleration of gravity,

$G_1$  and  $G_2$  – kinetic energy of the air stream in the inlet and measuring nozzles,

$w_{1sr}$  and  $w_{2sr}$  – mean velocity of the air in the inlet and measuring nozzles.

As it will be seen in the graphs of the experimental results below, the measuring nozzle head width  $d_o/d_p$  has a considerable impact on the dynamic properties of the air gauge.

The frequency response of a measurement system is found by a dynamic calibration [3]. In order to generate a sinusoidal input signal, the following equipment was applied. A shaft  $\varnothing 50.000$  mm with eccentricity  $e$  (Fig. 1) was placed in front of the measuring nozzle. When the shaft is rotating with rotational speed  $\omega$ , the measuring slot is changing its value  $s(\omega t)$  in a sinusoidal way from  $s - e$  to  $s + e$ .

The rotational speed of the eccentric shaft corresponding with the frequency  $f$  of the input signal was changed in order to obtain a frequency in the range from 0.1 to 20 Hz, with a step of 0.2 Hz.

In the experiments, the pressure  $p_k$  was registered by an oscilloscope (Fig. 2a) and further processed (Fig. 2b). In Fig. 2b values of  $A$  are presented as differences between the actual back-pressure  $p_{ki}$  and its mean value  $p_{ksr}$ :  $A_i = p_{ki} - p_{ksr}$ . The frequency and amplitude of pressure  $p_k$  were calculated as a mean value from 10 periods for each value of rotational speed  $\omega$  of the eccentric shaft. As the result a graph of the amplitude-versus-frequency was obtained. In Fig. 3, there are presented examples of obtained graphs for set D5 (*i.e.* with inlet nozzle  $d_w=0.625$  mm, outlet  $d_p=1.2$  mm, and ratio  $d_o/d_p=1.5$ ), where the amplitude is in absolute values (Fig. 3a) and related to the static amplitude  $A_0$  (Fig. 3b). After obtaining the information on amplitude versus frequency, the frequency  $f_{0.05}$  ensuring the dynamic error  $\delta(\omega)$  smaller than 5% could be determined. In the case shown in Fig. 3,  $f_{0.05} = 2.89$  Hz. However, the exact determination of frequency  $f_{0.05}$  requires some additional analysis.

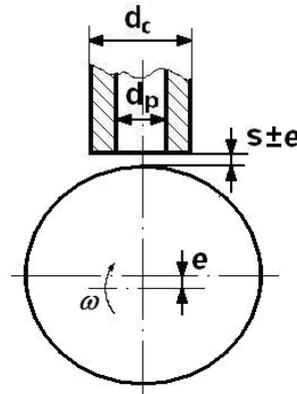
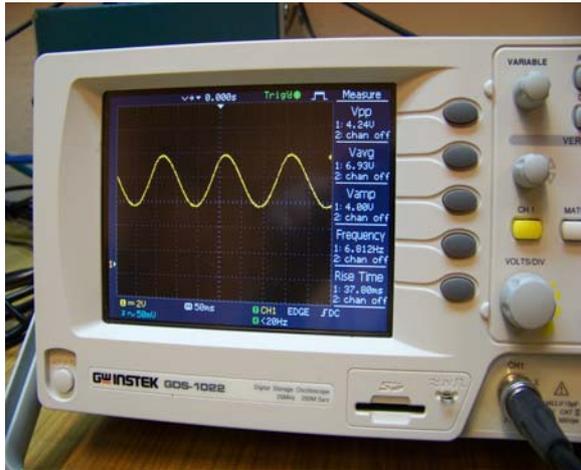


Fig. 1. The sine function input.

a)



b)

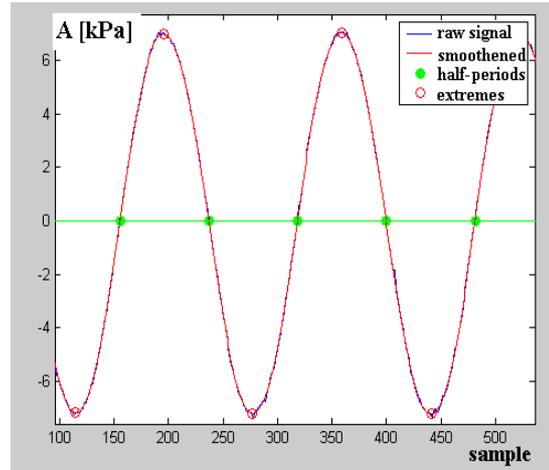
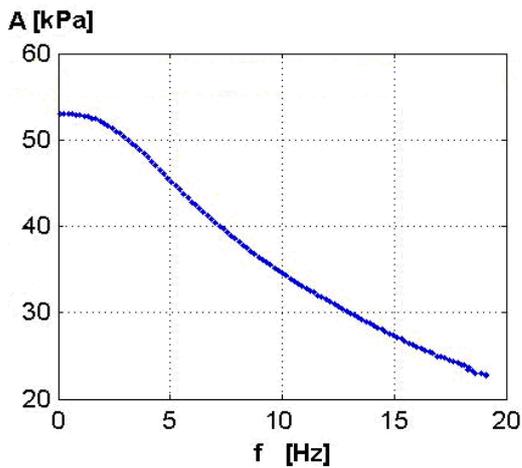


Fig. 2. An example of pressure  $p_k$  in the measuring chamber: a) registered by oscilloscope, b) processed by computer.

a)



b)

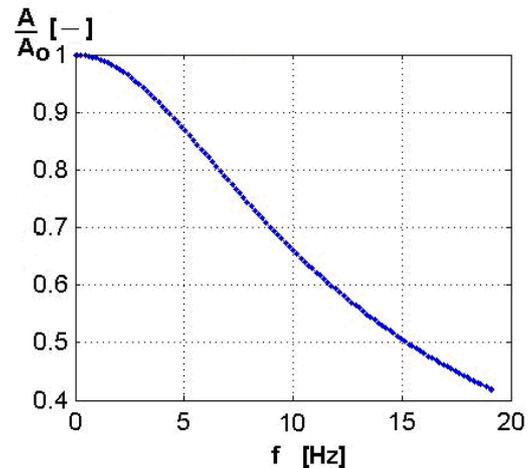


Fig. 3. Example of the obtained amplitude-frequency characteristics: a) absolute values, b) related to the static amplitude  $A_0$ .

The time constant  $T$  was calculated as well, using the procedure proposed by Soboczynski [11] and described in detail in [12]. Fig. 4 presents the block diagram of the experimental setup used for the dynamic calibration.

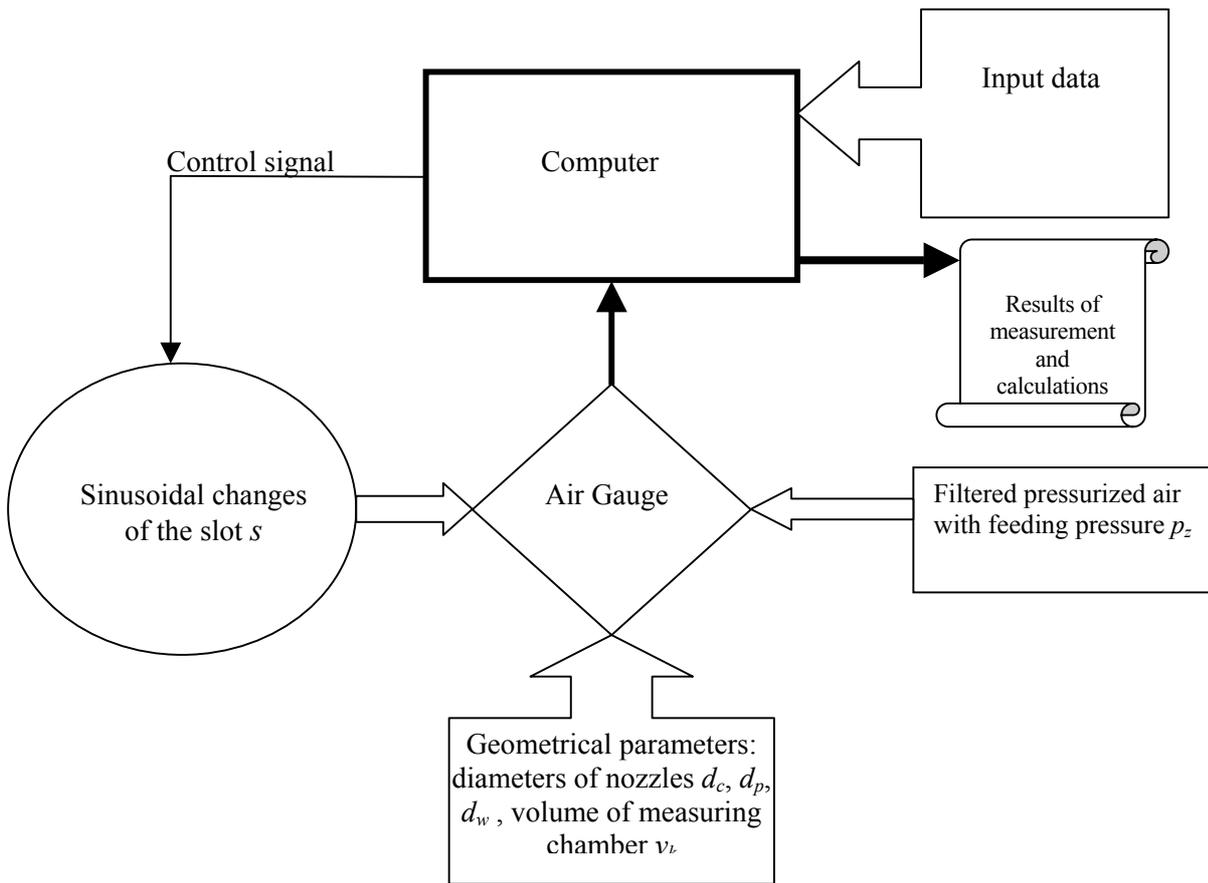


Fig. 4. Functional scheme of the dynamical calibration setup.

### 3. Repeatability test

The measurement in the described laboratory set bears a lot of uncertainties connected with pressured air, its temperature and pressure, temperature and pressure of air in the laboratory, measuring and control devices, calculation algorithms and mechanical inaccuracy. It seems reasonable to assume that a vast majority of these factors causes variation of the final results, so type A uncertainty evaluation is appropriate for this measurement. Therefore, in order to evaluate the uncertainty of a single experiment aimed to determine the time response  $T$  of the examined air gauge, a series of repetitions were undertaken with supervision of repeatability conditions.

The repetitions were performed for the air gauge with measuring nozzle  $d_p=1.20$  mm,  $d_c/d_p=3.0$ , inlet nozzle  $d_w=1.20$  mm and the measuring chamber no. 9 (volume  $v_{k9} = 3.921$  cm<sup>3</sup>). The examined set marked D6 was the one presenting the most unstable performance while examining static characteristics, so it should be expected that it would present the highest level of variation of results. In the experiments, the static amplitude of back-pressure generated by the rotating eccentric shaft was  $A_0 = 12.74$  kPa. It was assumed that because each result of measurement is based on 10 periods of sine function input, there is no need to perform 50 repetitions of full cycle measurement, especially as it would take a very long time. Table 2 presents the obtained results of 10 repetitions:

Table 2. Results of the repeated measurements.

No	1	2	3	4	5	6	7	8	9	10
$T$ [s]	0.006	0.007	0.006	0.006	0.006	0.005	0.008	0.006	0.008	0.006

From the obtained results, the mean value and the standard deviation could be calculated, which are as follows:

$$\bar{T} = 0.0064 \text{ s}$$

$$u(T) \approx s_T = 0.00097$$

Hence, provided each particular value of time constant  $T$  was determined as a mean value from 10 measurements repeated 10 times, its expanded uncertainty should be calculated with the coverage factor based on Student's distribution, which for the confidence level  $p = 0.95$  and 10 repetitions is  $t_{\alpha,n} = 2.262$ :

$$U_{0.95T} = 2.262 \cdot u(T) = 0.0022 \text{ s},$$

In order to evaluate the repeatability of the measurement system, equipment variation  $EV$  was calculated according to Procedure 3 [13]. Here, the measurement was repeated 10 times for 2 different air gauges. Next, the equipment variation was calculated as follows

$$\sum E = \sum_{i=1}^n \sum_{j=1}^k (X_{ij} - X_{i\bullet})^2, \quad (4)$$

where:

$X_{i\bullet}$  - mean value for the particular air gauge,

$i$  - number of examined air gauges, from 1 to  $n$ ; here  $n=2$ ;

$j$  - number of repetitions, from 1 to  $k$ ; here  $k=10$ .

The results of calculations are presented in Table 3.

Table 3. Results of the variation analysis.

$j$	1	2	3	4	5	6	7	8	9	10	$X_{i\bullet}$	$\sum_{j=1}^k (X_{ij} - X_{i\bullet})^2$
$T_{1j}$	0.004	0.003	0.004	0.003	0.003	0.004	0.004	0.003	0.002	0.004	0.0034	0.0000044
$T_{2j}$	0.006	0.007	0.006	0.006	0.006	0.005	0.008	0.006	0.008	0.006	0.0064	0.0000084
												$\sum E = \mathbf{0.0000128}$

Then equipment variation could be calculated from the formulas

$$s_E^2 = \frac{1}{n(k-1)} \sum E \quad (5)$$

and

$$EV = 5.15s_E \quad (6)$$

for a confidence level of 99%. The parameter  $EV$  calculated this way is  $EV = 0.0043$ .

The equipment variation appears to be very large. It should be noted, however, that its value covers all sources of uncertainty, and one of those sources is instability of the measuring signal caused by air flow-through phenomena [4]. Hence, despite the method itself provides repeatable experiments and calculations, some of the results could appear as far from the expected value as the parameter  $EV$  bounds. In general, however, main trends and relations in dynamic characteristics of the air gauges could be well identified.

The additional repetitions were performed for a larger measuring chamber ( $v_{kg} = 3.921 \text{ cm}^3$ ) and extended volume of the feeding pipe. A larger measuring chamber was chosen because it provides larger values of the time constant and in final results the difference caused by the feeding pipe would be revealed to a higher degree. In the first experiment, the air gauge had a pipe of 500 mm length and  $\varnothing 5$  mm, and in the second one the volume of  $500 \text{ cm}^3$  was added. Table 4 presents the results of measurements and calculations.

Table 4. Results of the experiment with extended pipe.

Shorter feeding pipe	Extended feeding pipe
$\bar{T} = 0.0057 \text{ s}$	$\bar{T} = 0.0064 \text{ s}$
$u(T) \approx s_T = 0.00082$	$u(T) \approx s_T = 0.00097$
$U_{0.95T} = 2.262 \cdot u(T) = 0,0019 \text{ s}$	$U_{0.95T} = 2.262 \cdot u(T) = 0,0022 \text{ s}$

The results presented in Table 4 clearly show that there is certain difference in the time constants of the air gauges dependent on the feeding pipe length or volume. None of the published paper points this problem out. It may be so because in most publications the behavior of larger measuring chambers underwent examination, which is less sensitive to the length of feeding pipe between stabilizer and inlet nozzle. It should be noted, however, that the differences between mean values lay in the area of uncertainty, so in many cases it could be omitted, especially when the relation between the volume of the feeding pipe and the volume of the measuring chamber is much smaller. In the present experiment the added volume was 125 times larger than the measuring chamber. Moreover, a further thorough investigation of the phenomena and its influence requires more accurate devices and setup.

#### 4. The result of measurement

The time constants  $T$  were determined for 9 different volumes of measuring chambers. In general, it could be stated that the configurations of the air gauges specified above in Table 1 in the investigated range of measuring chamber volumes never exceeded the time constant of  $T=0.021$  seconds. In most cases, as it was expected, the larger volume of the measuring chamber generated a longer time constant. On the other hand, the air gauges with higher sensitivity had a larger time constant than the ones with smaller sensitivity (for the same volume of the measuring chamber).

Fig. 5 presents the graph of time constants obtained for the air gauges with smaller measuring nozzle heads and wider ones. The smallest number of the measuring chamber corresponds to volume  $v_{k1} = 0.251 \text{ cm}^3$ , and the largest to  $v_{k9} = 3.921 \text{ cm}^3$  respectively. The indexes of the air gauge configurations are explained above in Table 1.

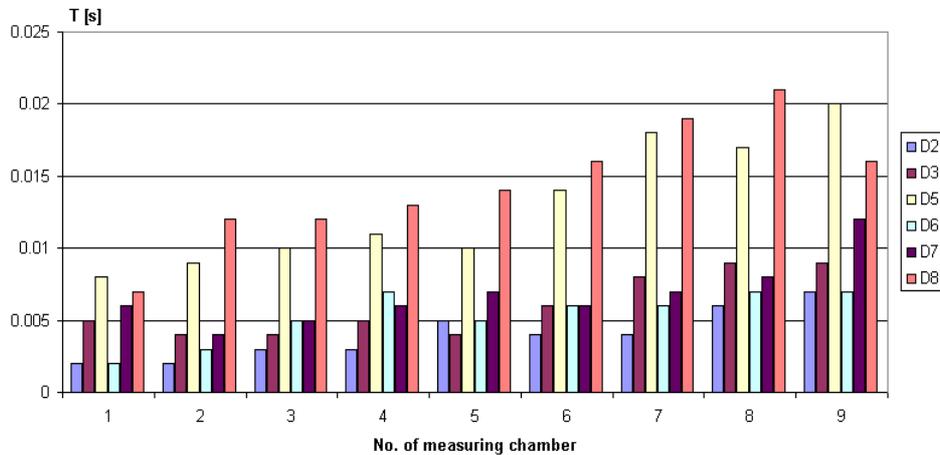


Fig. 5. The time constants achieved for different volumes and different air gauges.

In Fig. 5 it is seen that generally the time constant  $T$  is a little larger for the gauges with wider nozzle head, namely  $d_c/d_p = 3.0$  (D6, D7, D8). The variation of the presented values could be explained to a large extent by the instability of air flow in the measuring chamber described in [9], which caused also the substantial decrease of the measuring range  $z_p$  in case of D6. Those phenomena stay in close connection with measuring nozzle width. Similarly, the value  $d_c/d_p$  has its impact on the time constants. To make it more clear, an example of comparison of  $T$  values for  $v_{k5} = 1.224 \text{ cm}^3$  is given in Fig. 6a. It should be noted, however, that the impact of the nozzle head dimension depends on the sensitivity of the actual air gauge, as it is seen in Table 1 in the case of static characteristics.

Additionally, the measurement of time constant was performed in different areas of the measuring range  $z_p$ : in the middle, at the beginning (smaller slot and higher pressure) and in the end parts. As it was stated in the paper [12], the difference should have been expected. The experiments performed for different volumes of the measuring chamber revealed that the time constant was smaller when the slots were smaller. Fig. 6b shows the results of  $T$  measurement for the set marked D2 above in Table 1. In every instance, the time constant determined for the end part of the static characteristics  $T_e$  (larger slot and lower pressure) was about 50% longer than that for the beginning  $T_b$ .

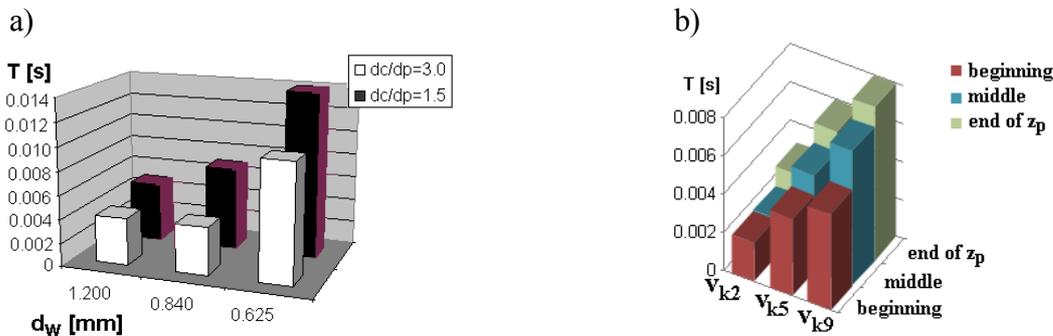


Fig. 6. Examples of the time constant obtained for the air gauge with  $d_p=1.200$  mm: a) for different  $d_c/d_p$  and  $d_w$ , b) in different areas of measuring range  $z_p$ , for  $d_w=1.200$  mm.

It should be noted that formula (3) proposed by [10], and other similar formulas given in [5] and [8] suggest that the time constant  $T$  is proportional to the volume of the measuring chamber  $v_k$ . It is difficult, however, to derive a proportion from the catalogue data of some producers. For example, [14] states that when the length of the pipe (which is a part of the measuring chamber in that solution) is 1 m, the time constant is  $T=0.3$  s, but when the length is extended to 2 m, the time constant almost doubles,  $T=0.5$  s. In that case the inner volume of the measuring head is unknown, so it is impossible to conclude what kind of proportion was applied, was it just an approximation from known formulas or experimental data. In the presented investigations, the proportion of the examined volumes is  $v_{k9} : v_{k5} : v_{k2} = 9.7 : 3.1 : 1.0$  while the proportion of the time constant is different:

- beginning of the measuring range  $z_p$ : 2.5 : 2.0 : 1.0,
- middle of  $z_p$ : 3.5 : 2.5 : 1.0,
- end of  $z_p$ : 2.6 : 2.0 : 1.0.

Similarly, for the same chambers, other air gauges show different proportions. To give just one example, the set marked D6 (Fig. 6a) has its time constants in proportion  $v_{k9} : v_{k5} : v_{k2} = 2.0 : 1.7 : 1.0$ . To explain that, some additional investigations should be performed and more exact formulas should be proposed.

## 5. Conclusions

Dynamical properties of the air gauge are very important in the inspection process. The examinations of the dynamical characteristics are of great practical importance, especially in case of devices designed for non-contact form deviation measurement (roundness, cylindricity) or profile analysis. As a result of investigations, it was proved that the time constant of the pneumatic system could be reduced down to several milliseconds. It should be noted that the dynamical properties of the air gauge depend not only on the flow-through geometry and the volume of the measuring chamber, but also on the pressurized air feeding system and its length (volume) and on the outer diameter of the measuring nozzle. The last parameter should not be ignored because of its large impact on the static characteristics as well.

When the dynamic properties of the air gauge are a priority, none of the constructional changes have such high effect as the introduction of the piezoresistive pressure sensors and the reduction of the measuring chamber. However, the proportion between the measuring chamber volume and time constant is not as simple as suggested in literature, at least for small chambers

Apart of commonly known relations between the dynamical characteristics of air gauges and feeding line length, and desired minimization of the measuring chamber volume, some new recommendations could be formulated based on the obtained results. First of all, when designing the air gauge with certain static characteristics it is possible to obtain almost similar parameters with different sets of inlet and measuring nozzles. It should be taken into consideration, however, that the air gauges with higher sensitivity have a greater time constant than the ones with smaller sensitivity.

Next, the air gauges with comparable nozzle diameters may reveal different dynamic performance because of a different outer diameter of the measuring nozzle (smaller nozzle head). In case of static characteristics, a smaller head surface causes an increase of the flow resistance coefficient and some loss of sensitivity. In dynamic characteristics, it takes effect with shortening of the time constant.

Additionally, it was proved experimentally that the time constant of the air gauge is dependent on the actual back pressure and differs in different areas of the measuring range. It is an important finding, especially as the difference could reach even 50%. In further investigations this problem should undergo a thorough examination because of its large impact on the precision of measurements with air gauges.

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