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GABOR TRANSFORM, SPWVD, GABOR-WIGNER TRANSFORM AND WAVELET TRANSFORM – TOOLS FOR POWER QUALITY MONITORING

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Abstract

The one-dimension frequency analysis based on DFT (Discrete FT) is sufficient in many cases in detecting power disturbances and evaluating power quality (PQ). To illustrate in a more comprehensive manner the character of the signal, time-frequency analyses are performed. The most common known time-frequency representations (TFR) are spectrogram (SPEC) and Gabor Transform (GT). However, the method has a relatively low time-frequency resolution. The other TFR: Discreet Dyadic Wavelet Transform (DDWT), Smoothed Pseudo Wigner-Ville Distribution (SPWVD) and new Gabor-Wigner Transform (GWT) are described in the paper. The main features of the transforms, on the basis of testing signals, are presented.

Keywords: Discreet Dyadic Wavelet Transform (DDWT), Gabor-Wigner Transform (GWT), Multiresolution Analysis (MRA), Power Quality (PQ), Smoothed Pseudo Wigner-Ville Distribution (SPWVD).

1. Introduction

The issue of power quality appeared at the beginning of the 20th century. It was the result of problems caused by the 3rd harmonic and interference of signals in close proximity of phone wires [1]. Thus, the higher harmonics theory was developed and the application of LC filters in power networks began. Further development of automatic and power electronic engineering caused an increase of the number of electronic converters in the power system, which were responsible for generating disturbances. Due to this, the power quality issue was defined.

The measurements of the power quality frequency parameters (*i.e.* THD factor) are currently performed with the help of FFT transformation [2, 3]. In spite of high computation efficiency, the method does not give positive results during measurements of fast spectrum changes. Therefore, current research is performed on the application of alternative methods enabling spectrum measurements and time localization.

Simultaneous localization disturbances in time- and frequency- domains can be performed with the help of time-frequency methods. Among many of time-frequency methods, monitoring of power quality parameters is taken into consideration: Wavelet Transform (WT) and Cohen's class analysis. In the Cohen's class different kinds of Wigner's transformations are used: Wigner-Ville Distribution (WVD), Pseudo Wigner-Ville Distribution (PWVD) and Smoothed Pseudo Wigner-Ville Distribution (SPWVD).

The SPWVD does not include characteristic distortions of WVD – time and frequency cross-terms. The SPWVD features loss of excellent WVD time-frequency resolution. To get better resolution than SPWVD and to avoid cross-term distortions, the Gabor-Wigner Transform (GWT) has recently been proposed. Actually, GWT is compilation of PWVD and Gabor Transform (GT) methods. It is an interesting comparison of: WT, GT, SPWVD and

GWT in detection of power supply disturbances and implementation of the methods in monitoring of power quality.

2. Power quality

Nowadays, the most frequently used methods in power quality measurements are frequency-domain analysis, especially Fourier transformation (in particular DFT). The main advantage of the method is small computation complexity. Thanks to this feature, it is possible to apply the method to inexpensive measurements system.

Discrete Fourier transformation (DFT), particularly fast algorithms (FFT) according to [3] are used to harmonics calculations as well as the total harmonic distortion factor THD for voltage signals. This method causes significant computational errors in case of existence of short-duration-time harmonics in the signal or when noise-character disturbances or impulses are in the signal. Average calculations of power quality parameters according to norm [2] for ten-minute intervals permit the minimization of these errors. In many cases they can even exceed 5 %. These calculations were made on the basis of good synchronization of the time window with the fundamental signal (the synchronization error was less than 50 ppm) or using a time window other than rectangular.

Hence, other and more advanced digital signal processing methods are currently considered for evaluating power quality parameters and for detecting disturbances which occur in power networks.

3. Time-frequency analysis

3.1. General considerations

For non-stationary signals (often encountered in power networks) research it is necessary to use time-frequency methods.

The methods can be divided in several ways. One of the divisions is shown in Fig. 1 [4, 5].
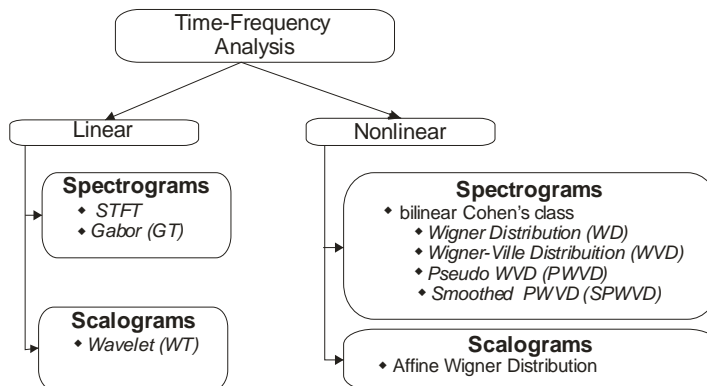

Fig. 1. One of the signal analysis classifications.

In linear methods, the signal being analyzed is compared directly with suitable elementary functions. The computational complexity of linear methods is relatively smaller in comparison to nonlinear methods.

Nonlinear methods have a significant advantage of direct energy signal projection on the time-frequency plane. This is particularly important in power signal measurements. A

disadvantage of linear, especially bilinear analyses, is the typical interference called cross-terms.

The so-called "transformation kernel", enabling a match of the analysis adaptation with the examined interference, should be properly chosen to reduce the said cross terms [5].

In the measurement of power quality parameters and power network disturbances the research is focused on application of multiple time-frequency methods that belong to the groups above. The most popular methods include: Short-Time Fourier Transform (STFT), Gabor Transform (GT) and Wavelet Transformation (WT). For bilinear transformation analysis there is a research on application of *i.e.*: Wigner-Ville Distribution (WVD), Pseudo Wigner-Ville Distribution (PWVD), Smoothed Pseudo Wigner-Ville Distribution, Choi-Williams Distribution (CWD).

There are also conducted researches concerning automatic classification of PQ disturbances based on RMS and FFT computations [6] and time-frequency analysis [7, 8].

3.2. Wavelet transform

One of the most effective time-frequency methods is the discrete dyadic wavelet transformation (DDWT). It is a prototype of continuous wavelet transformation which compares the examined signal with the so-called "little wave". This wavelet is scaled in frequency and dilated in time [5, 9]. The direct form of DDWT is not often used. Most often the wavelet decomposition of signal into detail coefficients Π_i and the approximation coefficients Ω_i are used (Fig. 2). The examined signal is being filtered by high-pass filtering (h_1) and then decimated which gives rise to detail coefficients. It contains high-frequency components existing in the analyzed signal. Details coefficients can be used for detection of short duration time disturbances (transients) on every decomposition level [5, 10]. Approximation coefficients originate in the same way, the difference is the use of low-pass filtering (h_0). Next the approximation coefficients are being decomposed according to the algorithm shown in Fig. 2.
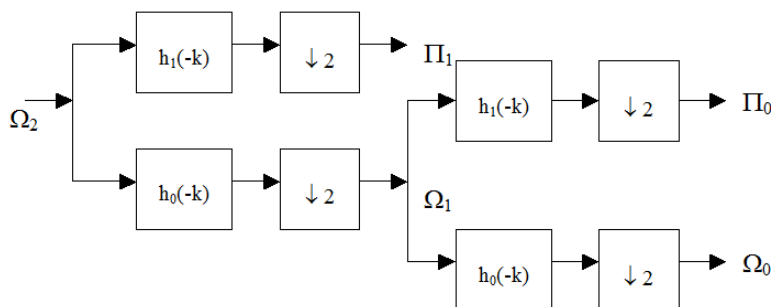

Fig. 2. Two decomposition levels of the DDWT (Ω_2 – the analyzed signal, Π_{1-0} – detail coefficients of the analyzed signal, Ω_{1-0} – approximation coefficients of the analyzed signal.

This decomposition can be continued till optimal time-frequency resolution is achieved. The pass-band center frequency decreases twice after every decomposition. Such a decomposition is called multiresolution analysis. Orthonormal wavelet filters are used in this decomposition. On the basis of detail coefficients it is possible to detect in time sags, overvoltages and other short-duration-time disturbances. Depending on the wave type it is possible to determine the time of their occurrence. Besides, every detail coefficients occurs in a defined frequency band which also localizes the disturbance in the frequency domain. The accuracy of time-frequency localization is restricted by the Heisenberg-Gabor principle which

is an equivalent of the Heisenberg uncertainty principle (1) [11]. In accordance with this principle the accuracy of localization in time and in frequency is described by the equation:

$$\Delta t_X \Delta \omega_X \geq 2\pi,$$ (1)

where:

- Δt_X – absolute error of localization in time;
- $\Delta \omega_X$ – absolute error of localization in frequency.

It is possible to decompose detail coefficients. This kind of decomposition is called wavelet packets [12, 13]. It is possible to calculate entropies and standard deviations for every detail coefficients and approximation coefficients. On the basis of their values the decision about next decomposition (the structure of the binary tree) is taken [12, 13]. There are a lot of families and kinds of wavelets which are used for decomposition. They differ in length and shape and as a result they differ in time-frequency properties. When DDWT is concerned, the frequency spectrum is the most important aspect since it influences detail coefficients and approximation coefficients.

3.3. STFT, Gabor Transform, SPWVD, Gabor-Wigner Transform

Originally, continuous Short-Time Fourier Transformation (STFT) was used for time-frequency representation of speech signals. It is a natural Fourier Transform extension with analyzing time-window overlay that enables to determine a point in time for signal spectrum fluctuation. The precision of such spectrum designation and its location in time depends on parameters such as: measurement window width, its shape, sampling frequency and application of measurement window overlapping. Optimal adjustment of parameters and results processing were the subject of many publications [14−17]. Also, guidelines for STFT analysis parameter selection were defined in standard PN-EN 61000-4-7 [3], which is currently one of the basic principles for designing electric power quality measuring devices.

The definition of such analysis is represented by formula (2) below [5]:

$$STFT(t, f) = \int_{-\infty}^{+\infty} s(\tau)\gamma^*(\tau - t)e^{-j2\pi ft}d\tau,$$ (2)

where:

- $s(t)$ − is a signal in the time domain;
- $\gamma(t)$ − is a signal in the time-window.

In particular, substitution of window $\gamma(t)$ (which is a Gauss function represented by formula (3)) to formula (2) results in (4):

$$\gamma(t) = e^{-\frac{1}{2}\left(\frac{t}{\sigma}\right)^2},$$ (3)

$$GT(t, f) = \int_{-\infty}^{+\infty} s(\tau)e^{-\frac{1}{2}\left(\frac{\tau-t}{\sigma}\right)^2}e^{-j2\pi ft}d\tau.$$ (4)

The Wigner-Ville transformation (distribution) WVD is presented as follows (5) [5]:

$$WVD_s(t, f) = \int_{-\infty}^{+\infty} s\left(t + \frac{\tau}{2}\right)s^*\left(t - \frac{\tau}{2}\right)e^{-j2\pi ft}d\tau,$$ (5)

where: $s(t)$ – investigated signal processed with the help of Hilbert Transformation (6):

$$\hat{s}(t) = \frac{1}{\pi} \int\limits_{-\infty}^{\infty} \frac{s(\tau)}{t-\tau} d\tau, \tag{6}$$

One of the advantages of WVD with relation to GT is resolution, which is twice as high. The transformation gives very good results (high time-frequency resolution) when the examined signals consist of a small number of higher harmonics. In other cases, the transformation results include interferences, the so called cross-terms. They appear between each pair of harmonics and make proper disturbance interpretation impossible. This is unacceptable for analysis of disturbed power signals. Currently research is conducted concerning methods of cross-terms reduction.

Another advantage of WVD is the fact that WVD gives direct information about time-frequency localization of signal energy. It enables the application of the transformation to evaluate energy included in higher harmonics and to localize it in the time domain.

Eq. (5) assumes a limit of integration for displacement from -∞ to +∞. As rule of thumb, such requirement is impossible to meet, so WVD is overlaid on the $h(t)$ time-window which results in Pseudo Wigner-Ville Distribution (PWVD) (as described by formula 7):

$$PWVD_s(t,f) = \int\limits_{-\infty}^{+\infty} s\left(t+\frac{\tau}{2}\right) s^*\left(t-\frac{\tau}{2}\right) h\left(\frac{\tau}{2}\right) h^*\left(-\frac{\tau}{2}\right) e^{-j2\pi f\tau} d\tau, \tag{7}$$

where: $h(t)$ – window reducing cross-terms it the time domain.

The time-window overlay operation is equal to WVD frequency filtering. Usually, the interference between time-shifted signals is attenuated, but cross-terms between frequency-shifted signals still exist.

There are several ways of resolving the problem of frequency cross-terms attenuation. Two methods are described below.

To minimize cross-terms between components in the frequency domain, PWVD results are attenuated with low-pass filtering, using the $g(t)$ attenuation window. Such analysis is described as Smoothed Pseudo Wigner-Ville Distribution (SPWVD), defined with formula (8):

$$SPWVD_s(t,f) = \int\limits_{-\infty}^{+\infty} g(t-t') \cdot PWVD_s(t',f) dt' =$$
$$= \int\limits_{-\infty}^{+\infty} h\left(\frac{\tau}{2}\right) h^*\left(-\frac{\tau}{2}\right) \int\limits_{-\infty}^{+\infty} g(t-t') s\left(t'+\frac{\tau}{2}\right) s^*\left(t'-\frac{\tau}{2}\right) dt' \cdot e^{-j2\pi f\tau} d\tau, \tag{8}$$

where:
- $h(t)$ – window reducing cross-terms in the time domain;
- $g(t)$ – window reducing cross-terms in the frequency domain.

The disadvantage of using filtering windows is limiting of original excellent time-frequency resolution features.

Selection of proper $h(t)$ and $g(t)$ windows in the expected interference function and requested spectrum resolution causes significant problems.

An alternative solution, to be used instead of SPWVD or other billing methods for cross-term reduction, is the Gabor-Wigner Transformation (GWT). GWT is defined by the following relations (9–12) [18, 19], and the detailed properties are shown in [19]:

$$GWT(t,f) = GT(t,f) \cdot WD(t,f), \tag{9}$$

$$GWT(t, f) = \min \left\{ |GT(t,f)|^2 \cdot |WD(t,f)| \right\}, \tag{10}$$

$$GWT(t, f) = WD(t, f) \left\{ |GT(t,f)| > 0.25 \right\}, \tag{11}$$

$$GWT(t, f) = GT^{2.6}(t, f) \cdot WD^{0.6}(t, f). \tag{12}$$

In fact, GWT is a composition of two time-frequency planes, being a result of Gabor Transformation and WVD. Such approach can be taken thanks to linked advantages of both transforms: excellent WVD time-frequency properties and lack of cross-terms – GT. Because of the real signal analysis it is not uncommon to use PWVD instead of WD.

4. Case studies

4.1. Wavelet transform

Many DDWTs were made for simulated voltage signals with disturbances (Fig. 3, Fig. 4, Fig. 6). In Fig. 3 and Fig 4. detail coefficients with the use of two wavelets db2 and db8 (Daubechies wavelet) have been presented. The analyzed signal consists of four different parts (sampling frequency F_s = 16384 Hz). Two parts with 50 Hz frequency and different amplitudes, the third part value equals zero and the fourth part frequency is 50 Hz with added higher harmonics (2 and 4) with the same amplitudes (see details in point 4.2.). In the first detail coefficients short impulses can be seen which show differences between the signals. Comparing the results of two wavelets it can be seen that for wavelet db2 the amplitude impulses are larger and time localization is more accurate. It results from the fact that wavelet db2 is attenuated more intensively and more quickly in comparison with wavelet db8 but wavelet db2 has a wider frequency band. In Fig. 5 the standard deviation and three different entropies of details coefficients for 12 decomposition levels of this signal have been presented. There are enormous differences between results for standard deviation and Shannon entropy and "log energy" entropy and "sure". The eighth decomposition level is equivalent to the fundamental frequency of signal. This can be seen for standard deviation and Shannon entropy, however the remaining two entropies are more sensitive to considerable amplitude changes.

Fig. 6 shows a fundamental power signal (50 Hz) with two disturbances (two exponentially attenuated sinusoidal signals of 250 Hz, F_s = 16384 Hz). Detail coefficients present starts of the disturbances and their duration time. Shannon entropy and standard deviation in Fig. 7 show that there are disturbances in detail coefficients number 2, 3, 4 (detail coefficients number 7, 8, 9 show the fundamental frequency).

Descriptors presented above can be used only for detection and preselection of disturbances. Only a more detailed analysis of detail coefficients can enable full classification of disturbances. In order to do this a reconstruction of the signal without approximation coefficients can be used. This reconstruction is being done by means of a complementary reconstruction wavelet filter and upsampling. This enables to separate disturbances and further analyses. An example of disturbance detection and its reconstruction is presented in Fig. 8. On the basis of detail coefficients the simulated disturbance has been detected and localized in time. In order to reconstruct impulse disturbance by means of Shannon entropy or standard deviation it is necessary to select appropriate detail coefficients and set all elements of approximation coefficients to zero. When these conditions are fulfilled, the reconstruction is being done.
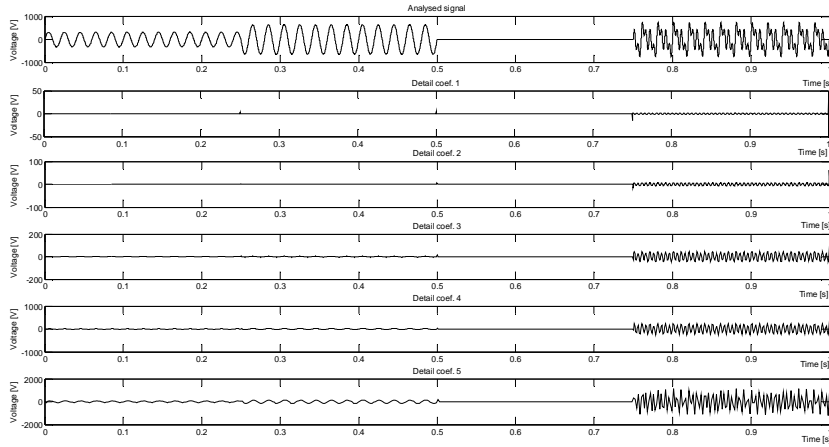
Fig. 3. The decomposition of a simulated signal with disturbances on detail coefficients
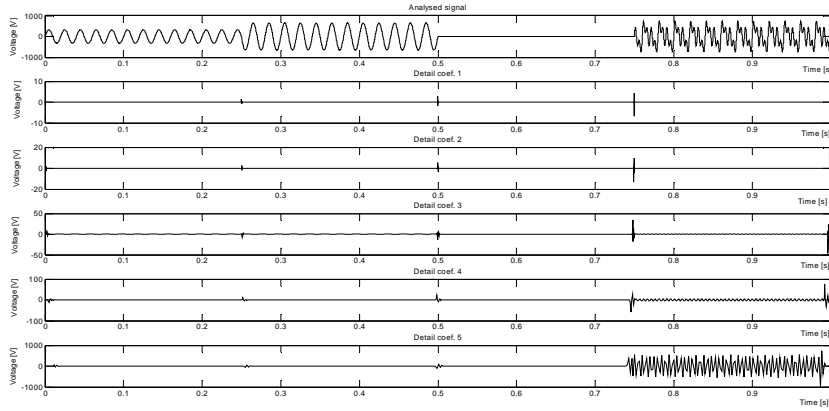(5 decomposition levels and wavelet db2).

Fig. 4. The decomposition of a simulated signal with disturbances on detail coefficients
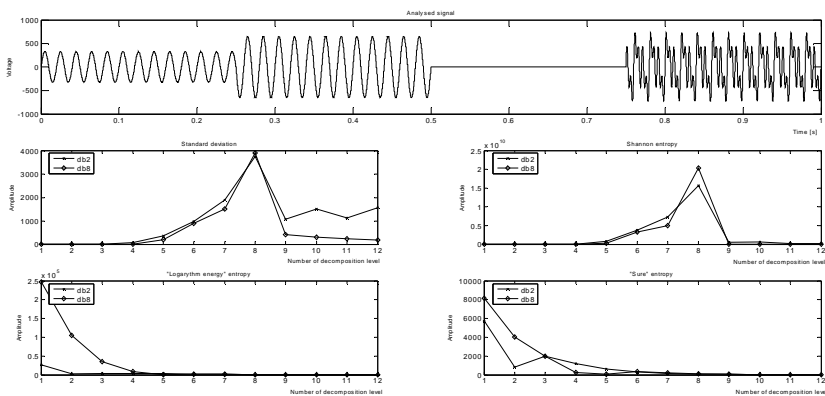(5 decomposition levels and wavelet db8).

Fig. 5. The simulated signal with disturbances and standard deviation and entropies of detail coefficients
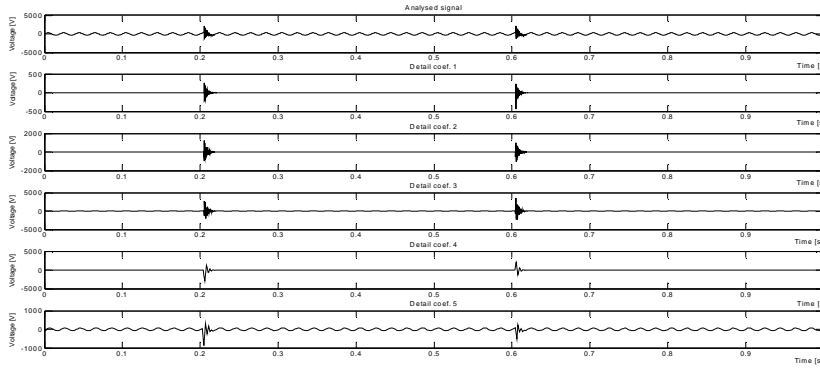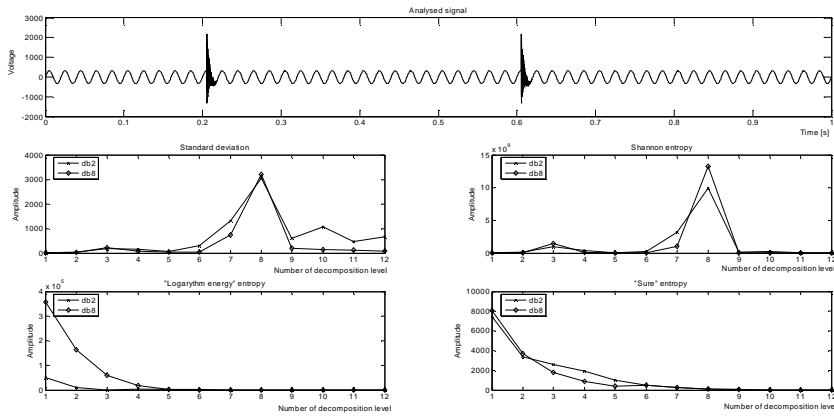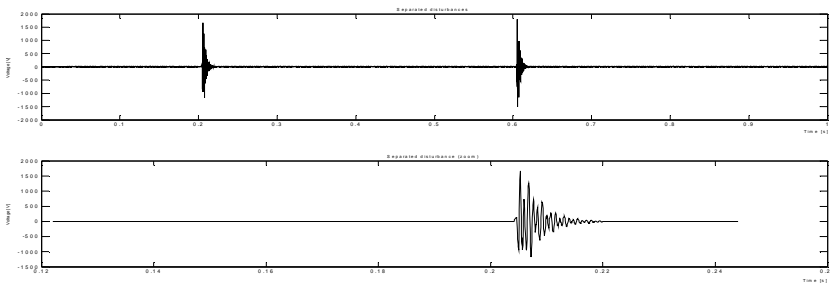(12 decomposition levels and wavelet db2 and db8).

Fig. .6. The decomposition of a simulated signal with disturbances on detail coefficients
(5 decomposition levels and wavelet db2).

Fig.7. The simulated signal with disturbances and standard deviation and entropies of details coefficients
(12 scales and wavelet db2 and db8).

Fig. 8. The reconstruction of detected disturbances in simulated signal (Fig. 6.)
(5 decomposition levels and wavelet db2).

4.2. GT, GWT, SPWVD

Voltage dips, voltage interrupts and harmonics are interferences often seen in power networks. To demonstrate the analysis reaction time for those phenomena, the model test signal was proposed (signal 1):

- 0 ms to 250   − 230·sqrt(2)·sin(2·π·50·t)
- 250 ms to 500 ms   − 2·230·sqrt(2)·sin(2·π·50·t)
- 500 ms to 750 ms   − 0
- 750 ms to 1000 ms   − 230·sqrt(2)·sin(2·π·50·t)+230·sqrt(2)·sin(2·π·100·t)+…
  …+230·sqrt(2)·sin(2·π·400·t)

The model of the test signal consists of a correct signal, overvoltage, interruption and harmonics. The GT, SPWVD and GWT analyses were examined . The following parameters of methods were chosen:

- GT − Gaussian window width 0.18 s ($\sigma = 0.021$ s);
- GWT from formula (14) – Gaussian window width 0.18 s ($\sigma = 0.021$ s), $h(t)$ − Hamming window (0.06 s);
- SPWVD − $h(t)$ and $g(t)$ Hamming windows (0.06 s).

Figs 9−11 present the time-frequency planes for GT, GWT and SPWVD respectively. The central sections of those images present the time-frequency plane, in the lower section there is a form of a course in time, the so called frequency-marginal condition is situated on the left, while the upper section contains the time-marginal condition. For better interpretation of the investigated signal a 3D view of GWT is shown in Fig. 12.

To emphasize the differences between results for individual methods, Fig. 13 shows a comparison of temporal sections (profiles) on the 50 Hz time-frequency plane and frequency sections (profiles) for a time of 0.9 s. In comparison to GT and SPWVD, GWT has a faster reaction to a signal change in time. What is more, in comparison to GT it has higher frequency resolution with SPWVD transformation. Moreover, GWT has very similar frequency resolution due to application of the same $h(t)$ window in both methods, but GWT has highest attenuation of frequency cross-terms.

To demonstrate the analysis reaction time for fast transients, a correct power signal was disturbed with the help of two exponentially attenuated sinusoidal signals of 250 Hz (signal 2). The time duration of both disturbances was 15 ms. The amplitude of the disturbances was 2000 V. Particular parameters of the methods were the same as in previous researches. Figs. 14-16 present time-frequency planes for GT, GWT and SPWVD respectively. Fig. 17 presents a 3D view of GWT results. Finally, Fig. 18 includes time (for 250 Hz) and frequency (for 0.20 s) profiles. Similar to the previous researches the best time-frequency localization of disturbances was for GWT, the worst, however, were for GT.
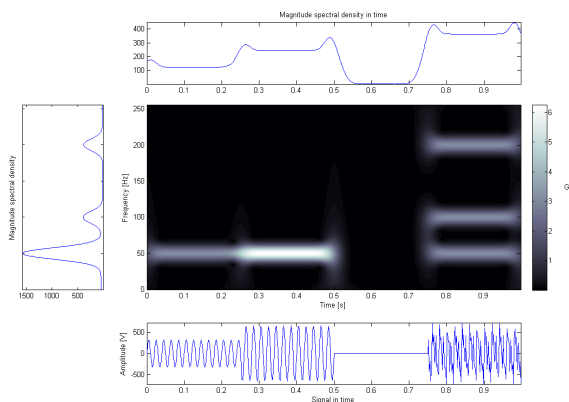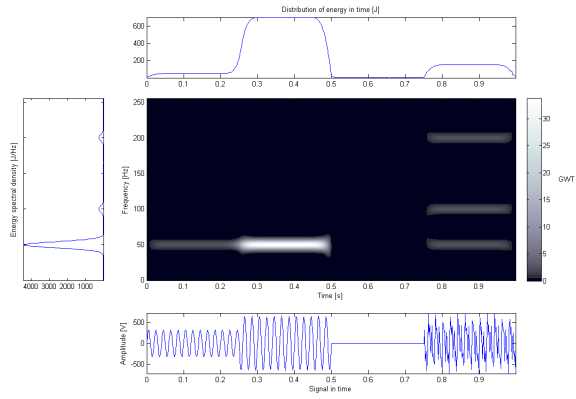

Fig. 9. GT time-frequency plane of signal 1.

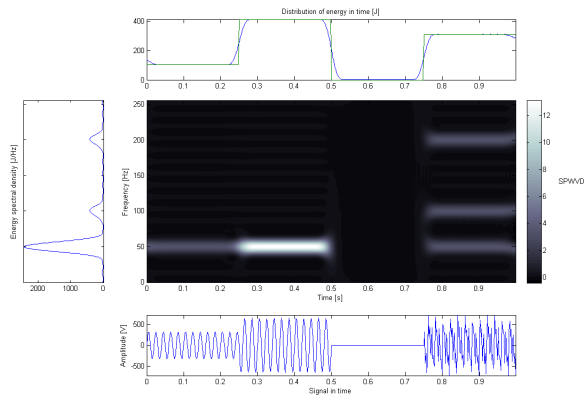Fig. 10. GWT time-frequency plane of signal 1.

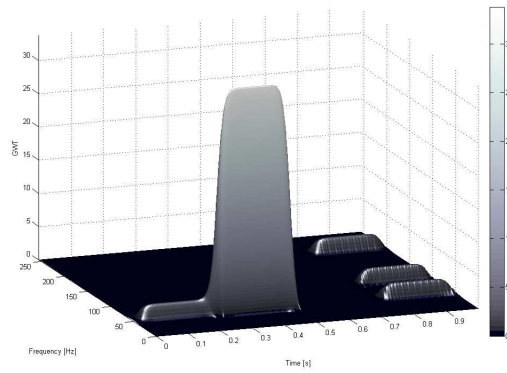Fig. 11. SPWVD time-frequency plane of signal 1.

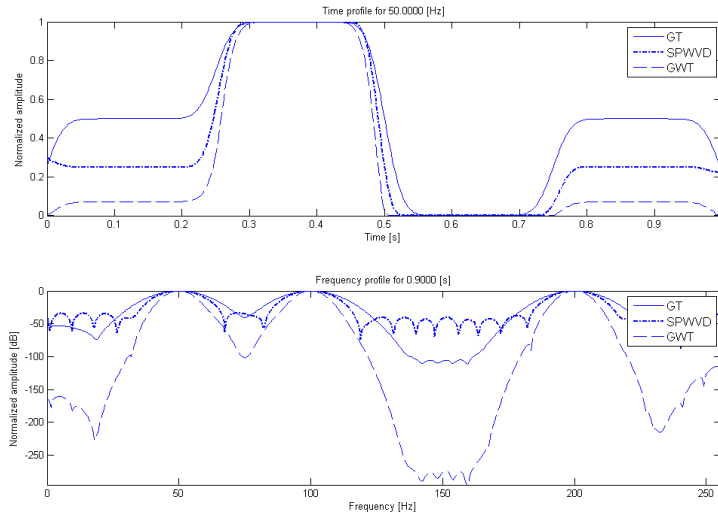Fig. 12. 3D view of GWT time-frequency plane of signal 1.

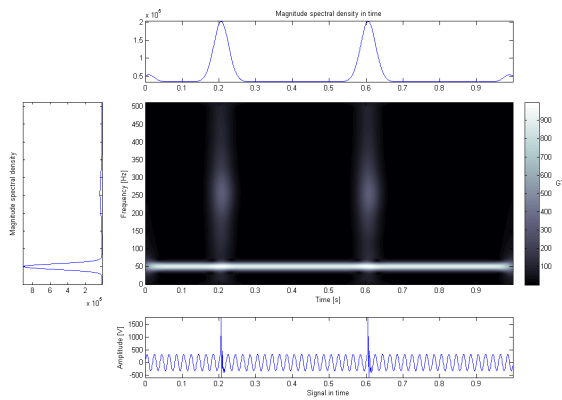Fig. 13. Time and frequency profile comparison for GT, SPWVD and GWT of signal 1.
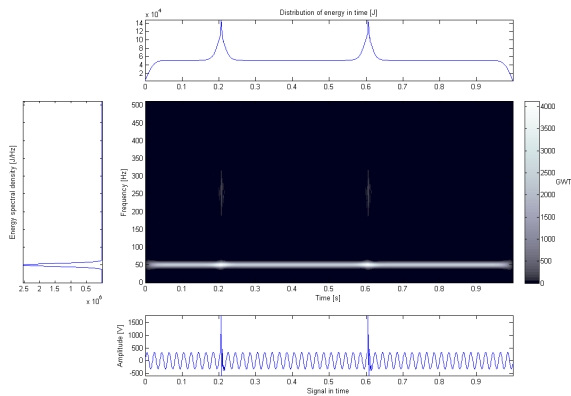
Fig. 14. GT time-frequency plane of signal 2.

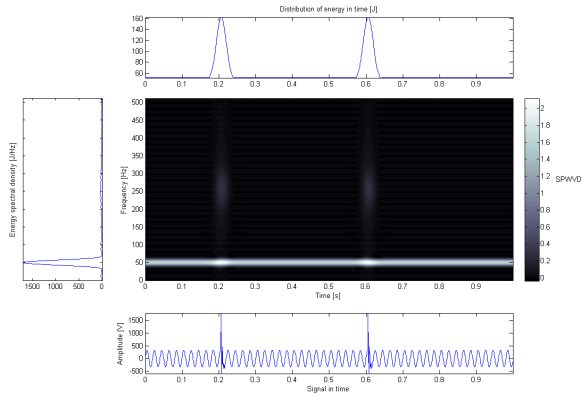Fig. 15. GWT time-frequency plane of signal 2.

Fig. 16. SPWVD time-frequency plane of signal 2.
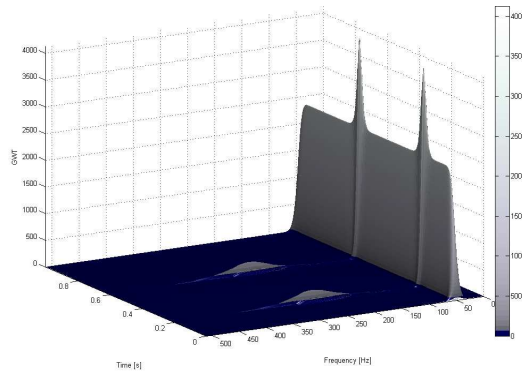

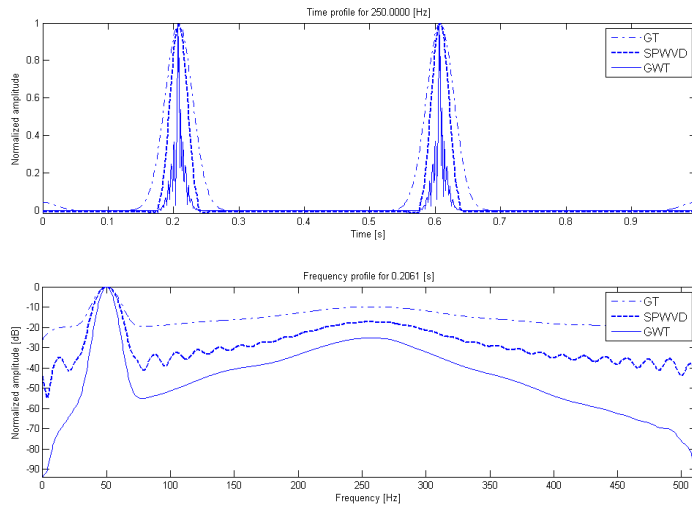Fig. 17. GT time-frequency plane of signal 2.

Fig. 18. Time and frequency profile comparison for GT, SPWVD and GWT of signal 2.

5. Conclusions

Several time-frequency tools for power signal analysis have been presented above. The possibilities of their use in the detection of disturbances and higher harmonics measurement have been described. It has been proved that DDWT is an effective method detecting various non-stationary signals. Further analysis using different entropies and standard deviation of detail coefficients (DDWT) enables the reconstruction and separation of many disturbances. The efficiency and relatively low processor capacity of DDWT (they are simple digital filters) signal analysis in real time is possible.

GT, SPWVD, as opposed to DDWT, enable direct energy projection in the time-frequency plane. Thus, it is possible to evaluate precisely the disturbance energy and to localize it in the time and frequency domains. This information simplifies disturbance identification and further diagnosis. In the example of a GWT analysis, the values on the time-frequency plane are not a direct projection of signal energy and it is necessary to perform further scaling of the evaluation results.

Rapid changes of the power signal are definitely more precisely detected with the help of DDWT. In the case of GT and GWT, the accuracy of time localization depends on the width of the Gaussian window and $h(t)$ window (only GWT), but in the case of SPWVD the smoothing window $g(t)$ is the most important.

In general, GT, SPWVD and GWT analyses are much more computationally complex than DDWT. That is why the methods can be applied in off-line analysis (postprocessing) to monitor the power supply and power quality.

Acknowledgements

References

[1] Z. Hanzelka: "Power Quality". *Międzynarodowa Konferencja jubileuszowa z okazji 50-lecia EAIE*, Kraków 7−8 Jun., 2002, pp. 65−68. (in Polish)

[2] EN 50160: 2007: *Voltage characteristics of electricity supplied by public distribution systems.*

[3] EN 61000-4-7:2007: "Electromagnetic compatibility (EMC)". *Testing and measurement techniques - General guide on harmonics and interharmonics measurements and instrumentation, for power supply systems and equipment connected thereto*, part 4−7.

[4] T.P. Zieliński: *Time-Frequency and Time-Scale Representation of Nonstationary Signals*. Wydawnictwa AGH, Kraków, 1994. (in Polish)

[5] A.D. Poularikas: *The Transforms and Applications Handbook*. 2nd ed., CRC Press, Boca Raton, 2000.

[6] Z. Ming, L. Kaicheng, H. Yisheng: "DSP-FPGA based real-time power quality disturbances classifier". *Metrol. Meas. Syst.*, vol. XVII, no. 2, 2010, pp. 205−216.

[7] M. Wang, A.V. Mamishev: "Classification of Power Quality Events Using Optimal Time-Frequency Representations – Part 1: Theory". *IEEE Trans. Power Deliver.*, vol. 19, no. 3, Jul. 2004, pp. 1488−1495.

[8] M. Wang, G.I. Rowe, A.V. Mamishev: "Classifiation of Power Quality Events Using Optimal Time-Frequency Representations – Part 2: Application". *IEEE Trans. Power Deliver.*, vol. 19, no. 3, Jul. 2004, pp. 1496−1503.

[9] C.K. Chui: *An introduction to wavelets*. San Diego, Academic Press, 1992, pp. 49−60.

[10] Y. Meyer: *Wavelets, Algorithms and Applications.* SIAM, Philadelphia, 1993.

[11] S.G. Mallat: "A theory for multiresolution signal decomposition: the wavelet representation". *IEEE Trans. Pattern Anal.*, vol. II, no. 7. Jul. 1989, pp. 674−693.

[12] A.M. Gaouda, M.M.A. Salama, M.R. Sultan, A.Y. Chikhani: "The Wavelet Representation A: Power Quality and Detection and Classification Using Wavelet-Multiresolution Signal Decomposition". *IEEE Trans. Power Deliver.*, vol. 14, no. 4, Oct. 1999, pp. 1469−1476.

[13] A. Cohen, I. Daubechies, B. Jawerth, P. Vial: "Multiresolution analysis, wavelets and fast wavelet transform on an interval". *CRAS*, vol. 316, Paris 1993, pp. 417−421.

[14] M. Szmajda, K. Górecki: "DFT algorithm analysis in low-cost power quality measurement systems based on a DSP processor". *9th International Conference Electrical Power Quality and Utilisation*, Barcelona Spain, 9–11 Oct. 2007.

[15] K. Górecki, M. Szmajda: "Adaptive digital synchronization of measuring window in low-cost DSP power quality measurement systems". *9th International Conference Electrical Power Quality and Utilisation*, Barcelona, Spain, 9–11 Oct. 2007.

[16] L. Satish: "Short-time Fourier and wavelet transforms for fault detection in power transformers during impulse tests". *IEEE Proceedings–Science Measurement and Technology*, vol. 145, no. 2, Mar. 1998, pp. 77–84.

[17] F. Zhang, Z. Geng, W. Yuan: "The Algorithm of Interpolating Windowed FFT for Harmonic Analysis of Electric Power System". *IEEE Trans. Power Deliver.*, vol. 16, no. 2, Apr. 2001, pp. 160–164.

[18] S.C. Pei, J.J. Ding: "Relations Between Gabor Transforms and Fractional Fourier Transforms and Their Applications for Signal Processing". *IEEE Trans. Signal Proces.*, vol. 55, no. 10, Oct. 2007, pp. 4839–4850.

[19] S.H. Cho, G. Jang, S.H. Kwon: "Time-Frequency Analysis of Power-Quality Disturbances via the Gabor-Wigner Transform". *IEEE Trans. Power Deliver.*, vol. 25, no. 1, Jan. 2010, pp.494–499.