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APPLICATION OF THE POLYNOMIAL INTERPOLATION METHOD FOR DETERMINING PERFORMANCE CHARACTERISTICS OF A DIESEL ENGINE

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Abstract

The article shows the methodology and calculation procedures based on Lagrange polynomial interpolation which were used to determine standard performance characteristics of the Polish production engine, type ANDORIA 4CTi90-1BE6. They allow to simplify the experimental research by maintaining a minimum number of measurement points and estimating the remaining data in an analytical way. The methods presented are convenient when it comes to the practical side because they eliminate the need for exploration of mathematical equations describing the various curves, which can be cumbersome and time consuming in the case of non-automated accounts. The results of analysis were applied to actual experimental results, indicating sufficient accuracy of the resulting approximations. As a result, procedures may be used in bench testing of a similar profile, especially with repeated cycles of the experiment, such as optimization of operating parameters of combustion engines.

Keywords: post measurements, polynomial interpolation, performance characteristics, diesel engine.

1. Introduction

The issue of interpolation primarily consists of finding, in a chosen interval, a function that accepts arbitrarily given values at certain points, so called nodes. Moreover, relatively the best approximation beyond them is expected, what is directly connected with the choice of the calculation method. It may be a prerequisite for the correct description of empirical data obtained from the experiment, whose potential disruption will lead to misinterpretation of the phenomenon in question. Interpolation as one of the basic numerical methods has been repeatedly used in studies of combustion engines, and the variety and complexity of the described processes determined specific procedures and calculation algorithms. For example, spline functions were used by Stotsky and Forgo [18] and they were used to estimate the acceleration of the engine crankshaft. Whereas Druault, Guibert and Alizon [5] used them to simulate the charge turbulence in the cylinder. In some works combining simple interpolation methods with other techniques can be found, in order to obtain the desired results. Erlandsson [6] modeled turbocharger performance with low-degree polynomials and extrapolation. Complex approximations procedures were presented in Vigild's work [21] but it was done earlier and to a much greater extent because it referred to the entire drive unit.

The problem of mathematical description of engine characteristics has already been taken up but a more complex methodology or different techniques were used for that purpose. For example in the scientific literature by Kudela, Jablonska, Shchetnikava, Karbach, Norling, Gerth, Nguyen, Zhou [13] for all experimental curves the approximation algorithms were used. Whereas, Wu, Fu, Wu [12] used low-degree polynomials but the procedure was related to make the universal characteristics. In the works of other researchers most often the linear or cubic interpolation and splines have been considered. This does not mean that the classical

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Lagrange formulas were not used but they related to separate issues such as: the assessment of individual performance of Suhail turbocharger [19], the simulation of Kim and Lee piston and valve travel [11] or discretization of measurement signals from Dobrolyubov engine [4].

Interpolation formulas in their classical form are much less common because they have become the starting point for deriving more complex methods. However, in the discipline of combustion engines, there are areas where they could be used not only in a much more frequent, but also effective way. That idea can be supported by the simplified calculation apparatus, since usually not only one, but very many interpolating functions are expected. Such a situation occurs while trying to describe curves that were obtained in the process of long-term operational tests, for repeated cycles of the experiment, e.g. at powering the engine with different fuel types, changing its regulatory setting or the correction of intake parameters, etc. The results are very often presented in a graphical form, as full power (performance) or partial power characteristics. As these courses are not especially complicated, the idea of limiting the number of measurement points which are needed for their determination, with the desired shape and course, and then estimation of the remaining data in the calculations has occurred. In principle, this procedure has to shorten the duration of the bench testing, while reducing operating costs. Elementary Lagrange interpolation was used for calculation analysis which was brought to the machine calculation comparison in three variants. Thus it is possible to conduct the current calculations without the laborious presentation of the final form of the polynomials but showing only the function values for earlier given points.

2. Basis of calculation apparatus

The starting point for calculations is the Lagrange interpolation formula which is applicable in the case of equal distances between the test points. The simplest form of this kind of polynomial has been cited by many authors in their works, e.g. Yang Cao Chung, Morris [23] or Fausett [7]:

$$W_N(x) = y_0 \frac{(x-x_1)(x-x_2)...(x-x_N)}{(x_0-x_1)...(x_0-x_N)} + y_1 \frac{(x-x_0)(x-x_2)...(x-x_N)}{(x_1-x_0)(x_1-x_2)...(x_1-x_N)} + \\ +...+ y_N \frac{(x-x_0)(x-x_1)...(x-x_{N-1})}{(x_N-x_0)(x_N-x_1)...(x_N-x_{N-1})}, \tag{1}$$

where:
$W_N(x)$ - Lagrange interpolation polynomial of N degree,
$x_0, x_1, x_2, \ldots, x_N$ - measurement points (nodes),
$y_0=f(x_0), y_1=f(x_1), y_2=f(x_2), \ldots, y_N=f(x_N)$ - function values at those points.

However, application of the equation (1) may be problematic, because increasing the number of nodes in a quick way leads to complexity of the calculations, what results from a high degree of the obtained polynomial. In case of hand-made calculations it is better to take advantage of iterative Aitken method, which guarantees the same approximation and increases the transparency of the recording. According to Fortuna, Macukow, Wąsowski [8], a first degree polynomial $W_{i,j}$, which at the nodes of x_i, x_j $(i \neq j)$ takes the value of y_i, y_j, can be presented with:

$$W_{i,j}(x) = \frac{\begin{bmatrix} y_i & x_i - x \\ y_j & x_j - x \end{bmatrix}}{x_i - x_j}. \tag{2}$$

Generally, it can be demonstrated by:

$$W_{0,1,\dots,k,m}(x) = \frac{\begin{bmatrix} W_{0,1,\dots,k-1,k}(x) & x_k - x \\ W_{0,1,\dots,k-1,m}(x) & x_m - x \end{bmatrix}}{x_m - x_k}. \tag{3}$$

This way the value of the interpolation polynomial at a given point, without its determining, can be calculated. The final result is the last part of the triangular matrix created, which is called the Aitken scheme:

$$\begin{bmatrix} x_0 & y_0 & & & & & & \\ x_1 & y_1 & W_{0,1} & & & & & \\ x_2 & y_2 & W_{0,2} & W_{0,1,2} & & & & \\ x_3 & y_3 & W_{0,3} & W_{0,1,3} & W_{0,1,2,3} & & & \\ x_4 & y_4 & W_{0,4} & W_{0,1,4} & W_{0,1,2,4} & W_{0,1,2,3,4} & & \\ \dots & \dots & \dots & \dots & \dots & \dots & & \\ x_N & y_N & W_{0,N} & W_{0,1,N} & W_{0,1,2,N} & W_{0,1,2,3,N} & \dots & W_{0,1,2,3,4,\dots,N} \end{bmatrix}. \tag{4}$$

In this variant, the next nodes are added very conveniently but the increase in their number may lead to deterioration of the results, as it is seen in the case of the classical Lagrange formula. This phenomenon usually occurs at the edges of the examined interval, and it is called the Runge's phenomenon. Its occurrence was described in detail in many publications, e.g. by Bjorck, Dahlquist [2], Yang Cao Chung, Morris [23], Cheney, Kincaid [3], Fortuna, Macukow, Wąsowski [8]. Most commonly it was manifested by the presence of extremes of the interpolating function, significantly deviating from the course of the analyzed curve. In such a situation more favorable results can be obtained by using the polynomials of lower degrees, e.g. of second or third ones, which was presented for the purpose of the hand-made calculation in the publication by Stoeck, Prajwowski [16]. Interval interpolation at the same time allows to maintain a constant distance between the nodes.

The presented models were used in practice to support the process of determining the performance characteristics of the Polish production engine, type ANDORIA 4CTi90-1BE6. As part of this article, three variants of calculation were compared: classical Lagrange formula, Aitken scheme and interval interpolation.

3. Testing equipment

The study was conducted on a typical engine dynamometer, the diagram of which was shown in Fig. 1. Operating parameters of ANDORIA 4CTi90-1BE6 engine were set by devices that were a part of the job, including, i.a.: AVL Dynoperform 160 eddy current dynamometer and AUTOMEX AMX 212F mass fuel gauge. The engine dynamometer was also equipped with complete control and measuring equipment, as well as systems ensuring the conditions to conduct research in accordance with guidelines of PN-ISO 15550:2009 [15].

Basic technical data of the tested engine is summarized in Table 1. This is the unit manufactured by Andoria-Mot Sp. z o. o. (Andoria-Mot Ltd.) with its registered office in Andrychów (Poland), primarily designed to drive big cars, SUVs and vans weighing up to

3.5 tons. The diesel oil summer "B", according to PN-EN 590 + A1: 2011 [14] was used as fuel. Measurements were made for the following engine speeds [rpm]: x_0=1400, x_1=1800, x_3=2200, x_4=2600, x_5=3000, x_6=3400. The results obtained, also summarized in Table 2, were used to designate the following performance characteristics: effective power $P_e=f(x)$, torque $T_{tq}=f(x)$, fuel consumption $B=f(x)$ and specific fuel consumption $b=f(x)$.

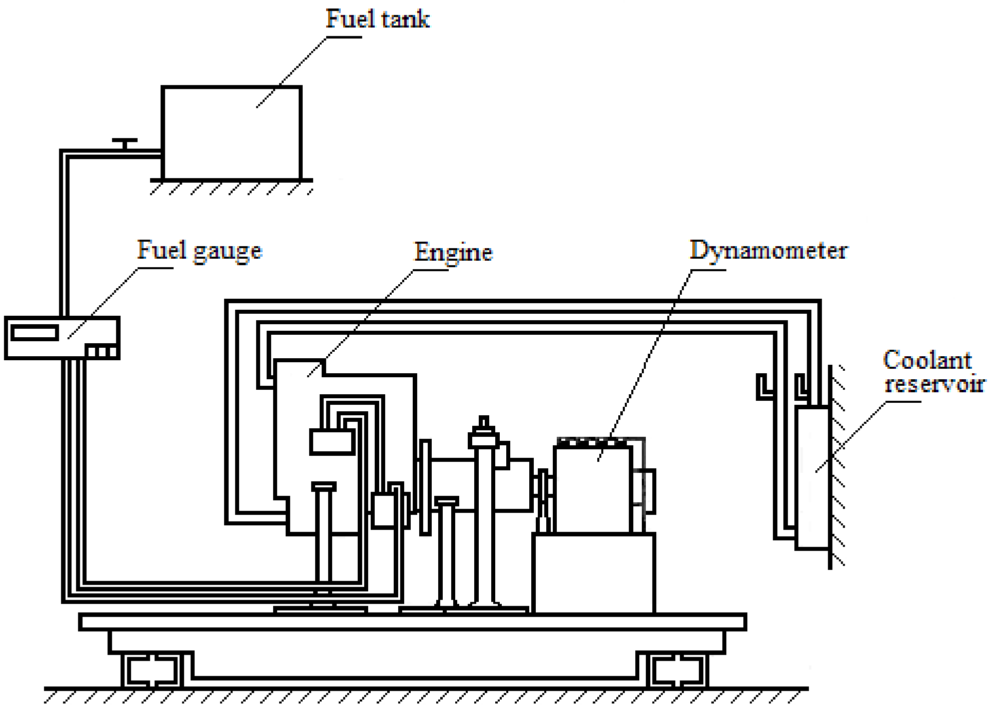

Fig. 1. Scheme of the test stand with the 4CTi90-1BE6 ANDORIA engine.

Table 1. Basic data of ANDORIA 4CTi90-1BE6 engine according to the manual [10].

No.	Parameter	Unit	Description
1	Type of engine	-	Diesel engine, four-stroke supercharged with intercooler
2	Type of charge	-	Radial turbocharger with exhaust waste-gate
3	Fuel injection	-	Indirectly to the swirl chamber
4	The order of injection	-	1-3-4-2
5	Injection pressure	[MPa]	15
6	Type of timing	-	Overhead valves timing with the camshaft located in the head
7	Number and arrangement of cylinders	-	4, vertical
8	Piston stroke	[mm]	95
9	Cylinder bore	[mm]	90
10	Engine cylinder capacity	[cm³]	2417
11	Compression ratio	-	20,6
12	Power rating	[kW]	66
13	Engine speed at power rating	[rpm]	4100
14	Maximum engine torque	[kN·m]	0,205
15	Engine speed at maximum torque	[rpm]	2000-2500
16	The minimum engine speed at idle	[rpm]	800
17	Specific fuel consumption at the time of maximum	[g/(kW·h)]	270

Table 2. The results of ANDORIA 4CTi90-1BE6 engine.

No.	x [rpm]	P_e [kW]	T_{tq} [kN·m]	B [kg/h]	b [g/(kW·h)]
1	1400	24,00	0,163	6,70	279,00
2	1800	36,50	0,192	9,40	257,42
3	2200	46,50	0,203	12,06	259,35
4	2600	53,90	0,199	14,40	267,16
5	3000	60,00	0,190	16,20	270,00
6	3400	64,00	0,180	18,00	281,25

4. Results and discussion

4.1. Application of Lagrange interpolation formula

First, the classical formula was applied (1); it was used to calculate the values of the function at points which are not the root nodes. The following values of intermediate argument were adopted: {x=1600, 2000, 2400, 2800, 3200}. In order to determine the final form of the polynomial interpolation $W_N(x)$, its separate components $S_{i=0,1,2,3,4,5}(x)$ had to be determined and summed up. The presentation of the calculation was performed for torque characteristics, indicating the sequence of selected actions.

Table 3. Determination of polynomial interpolation of the fifth degree for the argument x=1600.

i	x_i	y_i	$S_{i=0,1,2,3,4,5}(x)$	$W_N(x)$
0	1400	0,163	0,040	0,179
1	1800	0,192	0,236	
2	2200	0,203	-0,167	
3	2600	0,199	0,098	
4	3000	0,190	-0,033	
5	3400	0,180	0,005	

For example, the value of the first component in Table 3 was determined as follows:

$$S_{i=0}(x) = y_0 \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)(x_0-x_5)}. \tag{5}$$

After substitution of nodal points:

$$S_{i=0}(x) = 0{,}163 \frac{(x-1800)(x-2200)(x-2600)(x-3000)(x-3400)}{(1400-1800)(1400-2200)(1400-2600)(1400-3000)(1400-3400)}. \tag{6}$$

Hence for the argument $x=1600$ it was:

$$S_{i=0}(1600) = 0{,}040. \tag{7}$$

Summing up all the components allowed to determine the value of interpolating function at the point not being the cardinal node:

$$W_N(1600) = S_{i=0}(1600) + S_{i=1}(1600) + S_{i=2}(1600) + S_{i=3}(1600) + S_{i=4}(1600) + S_{i=5}(1600). \tag{8}$$

After substitution:

$$W_5(1600) = 0{,}040 + 0{,}236 + (-0{,}167) + 0{,}098 + (-0{,}033) + 0{,}005. \tag{9}$$

The final value of the fifth degree interpolating polynomial for x=1600 amounted to:

$$W_5(1600) = 0{,}179. \tag{10}$$

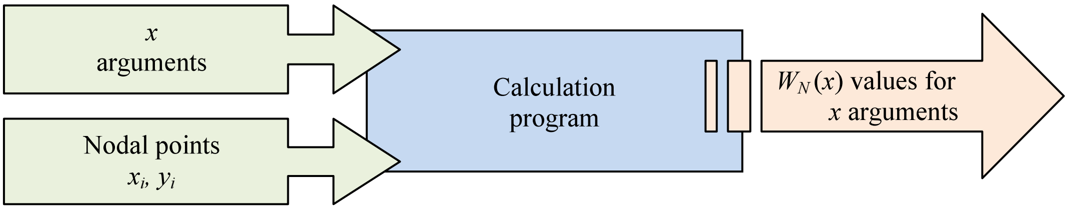

Fig. 2. The idea of calculating the value of polynomial interpolation at the given points of the discrete set.

A large number of components of the Lagrange polynomial makes the calculations tedious and time consuming. In accordance with the idea shown in Fig. 2, the efficiency of this process can be picked up in various ways, e.g. using available mathematical programs, writing a script in one of the computer languages or by entering the appropriate formulas in the spreadsheet.

Table 4. The results of experimental studies and analytical calculations.

No.	x [rpm]	T_{tq} [kN·m]	P_e [kW]	B [kg/h]	b [g/(kW·h)]
1	1400	0,163	24,00	6,70	279,00
2 [a)]	1600	0,179	30,41	8,05	264,38
3	1800	0,192	36,50	9,40	257,42
4 [a)]	2000	0,200	41,89	10,75	256,50
5	2200	0,203	46,50	12,06	259,35
6 [a)]	2400	0,202	50,44	13,29	263,56
7	2600	0,199	53,90	14,40	267,16
8 [a)]	2800	0,195	57,07	15,36	269,61
9	3000	0,190	60,00	16,20	270,00
10 [a)]	3200	0,185	62,49	17,02	272,31
11	3400	0,180	64,00	18,00	281,25

[a)] data estimated analytically

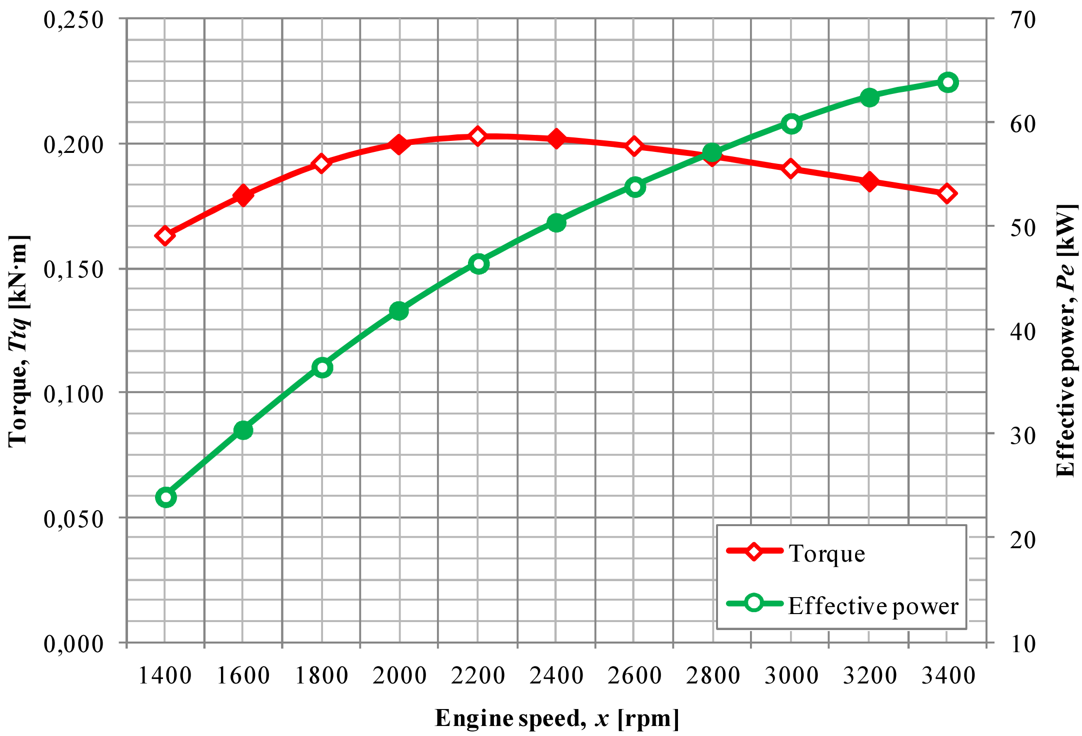

Fig. 3. Performance characteristics of ANDORIA 4CTi90-1BE6 engine: $T_{tq}=f(x)$, $P_e=f(x)$.

Examples of ready-made solutions and algorithms can be found in the scientific literature, e.g. by Bjorck, Dahlquist [2], Venkataraman [20], Billo [1], Fortuna, Macukow, Wąsowski [8], Wu, L., Fu, Z., Wu, W. [22]. Due to the numerical procedure chosen, calculation of the function value for other values of x does not present any difficulties. In this way, all the performance characteristics of the engine under test were set and the results are summarized in Table 4, detailing the data estimated analytically. Their graphical interpretation is shown in Fig. 3 and Fig. 4. In order to distinguish them, the main nodes (measured experimentally) were left unfilled, and the intermediate points (calculated) were completely filled with color.

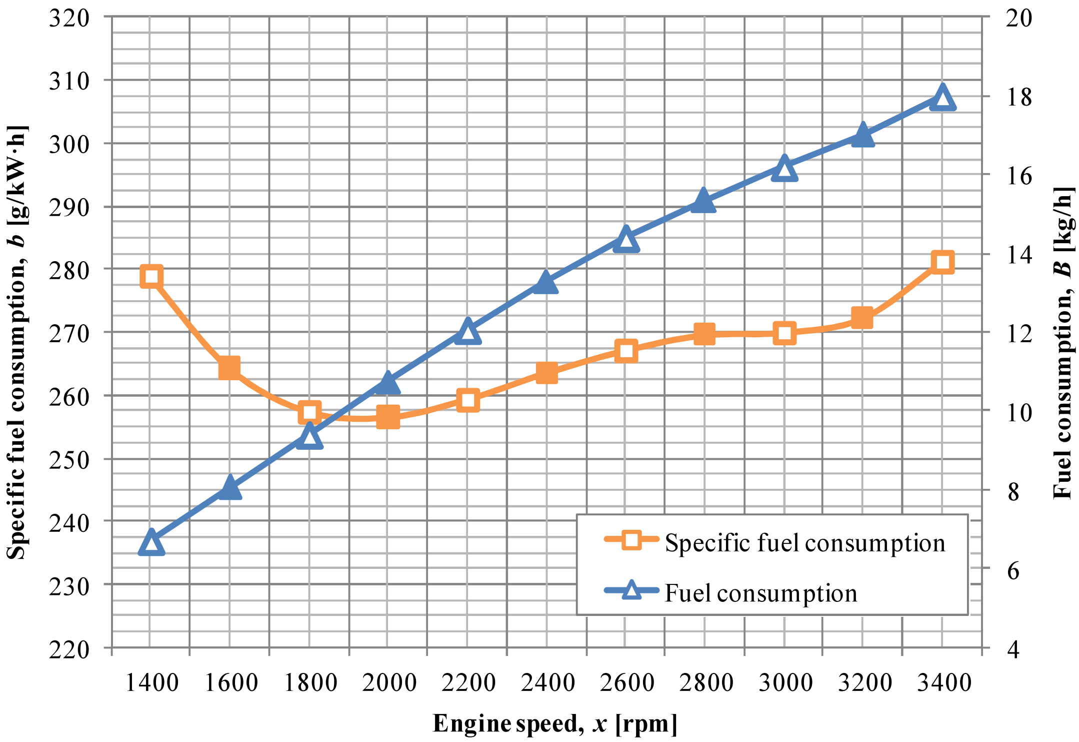

Fig. 4. Performance characteristics of ANDORIA 4CTi90-1BE6 engine: $b=f(x)$, $B=f(x)$.

4.2. The use of Aitken iterative method

Analogous results of the interpolating function approximation were obtained when the Aitken method was used (4). For example, for the data considered in Section 4.1, so for the torque characteristics, the triangular matrix will look as follows:

$$\begin{bmatrix} 1400 & 0{,}163 & & & & & \\ 1800 & 0{,}192 & 0{,}178 & & & & \\ 2200 & 0{,}203 & 0{,}173 & 0{,}180 & & & \\ 2600 & 0{,}199 & 0{,}169 & 0{,}180 & 0{,}180 & & \\ 3000 & 0{,}190 & 0{,}166 & 0{,}179 & 0{,}180 & 0{,}180 & \\ 3400 & 0{,}180 & 0{,}165 & 0{,}179 & 0{,}180 & 0{,}180 & 0{,}179 \end{bmatrix}. \tag{11}$$

In case of automatic calculations it can also be represented in the form of a table. However, irrespective of the manner of presentation, the calculation always uses the values lying on the left in the same row and above.

Table 5. Determining the values of polynomial interpolation for the argument x=1600.

x_i	y_i	$W_{0,i}(x)$	$W_{0,1,i}(x)$	$W_{0,1,2,i}(x)$	$W_{0,1,2,3,i}(x)$	$W_{0,1,2,3,4,5}(x)$
1400	0,163					
1800	0,192	0,178				
2200	0,203	0,173	0,180			
2600	0,199	0,169	0,180	0,180		
3000	0,190	0,166	0,179	0,180	0,180	
3400	0,180	0,165	0,179	0,180	0,180	0,179

For example, the last part of Table 5, that is the value of the polynomial interpolation $W_{0,1,2,3,4,5}(x)$ for the specified argument was calculated as follows:

$$W_{0,1,2,3,4,5}(x) = \frac{\begin{bmatrix} W_{0,1,2,3,4}(x) & x_5 - x \\ W_{0,1,2,3,5}(x) & x_4 - x \end{bmatrix}}{x_5 - x_4}. \tag{12}$$

After substituting components and nodal points:

$$W_{0,1,2,3,4,5}(x) == \frac{\begin{bmatrix} 0{,}180 & 3400 - x \\ 0{,}180 & 3000 - x \end{bmatrix}}{3400 - 3000}. \tag{13}$$

Hence for the argument x=1600 it was:

$$W_{0,1,2,3,4,5}(1600) = 0{,}179. \tag{14}$$

The convergence of the calculations carried out by the classical method and Aitken scheme makes that the results for all the external characteristics of the engine under test are identical, and they have been presented earlier (Table 4, Fig. 3 and Fig. 4). The correctness of the procedures can be easily checked, considering the situation for the value of the argument on any node (measuring point). For example, for $x=x_0$=1400 the final member of the matrix $W_{0,1,2,3,4,5}(x)$ must equal y_0 (Table 6).

Table 6. Checking the value of the polynomial interpolation for the argument x=1400.

x_i	y_i	$W_{0,i}(x)$	$W_{0,1,i}(x)$	$W_{0,1,2,i}(x)$	$W_{0,1,2,3,i}(x)$	$W_{0,1,2,3,4,5}(x)$
1400	0,163					
1800	0,192	0,163				
2200	0,203	0,163	0,163			
2600	0,199	0,163	0,163	0,163		
3000	0,190	0,163	0,163	0,163	0,163	
3400	0,180	0,163	0,163	0,163	0,163	0,163

4.3. Application of the interval interpolation

Using polynomials of lower degrees requires dividing the interval into smaller parts. It is very convenient for simple calculation processes which are not automated, but the automatic calculations do not pose much difficulty either. For the example considered in Sections 4.1 and 4.2, two polynomials were used: of the second and third degrees. Since the measuring point (x_2, y_2) was selected as the common node, thus for the argument $x=x_2$=2200 the values of both interpolating functions had to be identical. While analyzing performance characteristics of torque, the result of $W_{N=2}(x)=W_{N=3}(x)$=0,203 was obtained, which confirms the correctness of calculations with a single Lagrange polynomial (Table 4).

Table 7. Checking the value of the polynomial interpolation for the argument x=2200.

i	x_i	y_i	$S_{i=0,1,2}(x)$	$W_{N=2}(x)$	$S_{i=2,3,4,5}(x)$	$W_{N=3}(x)$
0	1400	0,163	0,000			
1	1800	0,192	0,000	0,203		
2	2200	0,203	0,203		0,203	
3	2600	0,199			0,000	0,203
4	3000	0,190			0,000	
5	3400	0,180			0,000	

The procedure is the same as in Section 3.1, but two groups of components determined separately for each polynomial are considered. Application of the procedure for all experimental data allowed to estimate the value of the function at points which are not the cardinal nodes (Table 8).

Table 8. Test results and analytical calculations for the interval interpolation.

No.	x [rpm]	T_{tq} [kN·m]	P_e [kW]	B [kg/h]	b [g/(kW·h)]
1	1400	0,163	24,00	6,70	279,00
2[a)]	1600	0,180	30,56	8,06	265,27
3	1800	0,192	36,50	9,40	257,42
4[a)]	2000	0,200	41,81	10,74	255,45
5	2200	0,203	46,50	12,06	259,35
6[a)]	2400	0,202	50,31	13,33	264,71
7	2600	0,199	53,90	14,40	267,16
8[a)]	2800	0,195	57,16	15,33	268,37
9	3000	0,190	60,00	16,20	270,00
10[a)]	3200	0,185	62,31	17,07	273,74
11	3400	0,180	64,00	18,00	281,25
[a)] data estimated analytically					

It should be noted here that in relation to the previous variants the results of approximations are slightly different. In the graphical interpretation the deviations can be observed only for the graph of specific fuel consumption, and thus for the performance characteristics of the least regular course (Fig. 4).

For the other curves, that is torque, effective power and fuel consumption, such a kind of presentation of results was dropped. Interpolating function values are in fact so similar that the determined characteristics overlap, giving a false picture of the identity of calculation procedures. In this situation, the choice of method is therefore much less important, supposing a small number of measurement points (at constant distances between them) will be maintained, and the interpolated graph of the function will not be marked by prominent failures, strongly outlined extremes, etc. If these conditions are not fulfilled, it can lead to oscillations of high degree polynomial, which is used for description at the whole interval.

4.4. Experimental verification of the results of interpolation

As shown in Fig. 5, better results of the approximations were obtained for the interval interpolation, since its curve almost perfectly applied to the description of $b=f(x)$, which was drafted only on the basis of experimental data. This does not mean that the use of single-fifth degree polynomial resulted in appreciably worse results. For example, at engine speed x=2800 rpm, the calculated value of specific fuel consumption amounted to b=269,61 g/kW·h (Table 4), being within measurement uncertainty for the experiment being carried out (b=268.60±2.66 g/kW·h). At the remaining points which were not cardinal nodes, comparable or even smaller oscillations were obtained. Therefore, approximation should be considered satisfactory if the maximum deviation allowed for the measurement of this parameter will be taken into account (±3% according to PN-ISO 15550:2009) [15].

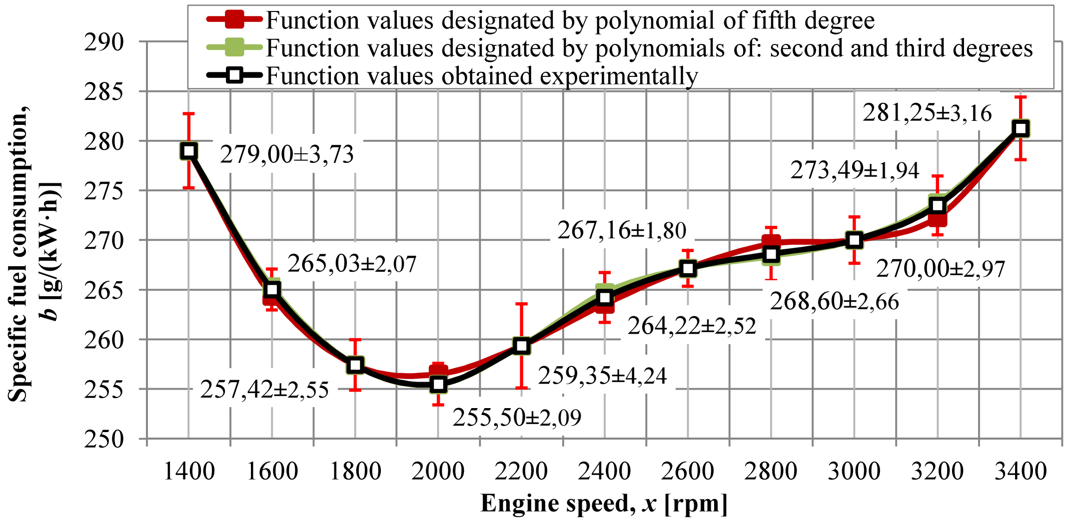

Fig. 5. Performance characteristics of specific fuel consumption, including the function values being obtained experimentally and indicating measurement uncertainties.

5. Conclusions

On the basis of the conducted analysis, it can be concluded that the calculation procedures presented here may be useful for determining the performance characteristics of the engine. Their use supports this process effectively, because estimation of data not contained in the plan of the experiment does not pose major difficulties from the technical point of view. For the machine calculation a change of input parameters (node, intermediate points) may be conducted online so each of the options under consideration can be easily adapted to similar studies in other positions. Moreover, they can be extended to those characteristics of an engine whose course is mild as well, e.g. partial power, operating power, smoke limit, etc.

The starting base for all procedures is the Lagrange interpolation formula, what is imposed by certain conditions necessary for their application. The first step is to maintain a constant distance between the measuring points at the experimental stage. Furthermore, adding more nodes can be problematic because the use of a single high degree polynomial over the whole discrete interval brings the risk of unwanted oscillations. This is related to the cases considered by the classical method and the Aitken scheme. Examples of interpolating function interference that describe the curves which have been designated in the research process for the other engine were presented in earlier publications by Stoeck, Prajwowski [16] and Stoeck [17]. In this case, it is safer to operate the interval interpolation, thus the third of the presented options, what additionally simplifies the calculation procedure. This method gives a better guarantee of accuracy of analytical estimates, and adjustment of the machine algorithm for low-degree polynomials is intuitive. Its correctness can be checked in a very easy way, using an arbitrary cardinal node, as it was shown in examples above.

More important abbreviations and marks

4CTi90-1BE6 - type of engine
AMX 212F - type of mass fuel gauge
b - specific fuel consumption

B - fuel consumption
i - numerically index of consecutive argument
N - degree of polynomial
P_e - effective power
$S_i(x)$ - components of polynomial
T_{tq} - torque
$W_N(x)$ - Lagrange interpolation polynomial of N degree
x - engine speed (arguments)
$x_0, x_1, \ldots, x_N$ - measurement points (nodes)
$y_0, y_1, \ldots, y_N$ - function values at measurement points

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